PF1.5: WORK, ENERGY AND POWER

Energy exists in many different forms, eg, kinetic energy E_k, potential energy U_g, electrical energy, and elastic (or spring) energy E_s. A fundamental principle of nature is that energy cannot be created or destroyed, only transformed or transferred.

Kinetic Energy KE

Kinetic energy KE, is energy associated with motion.

$$KE = \frac{1}{2} mv^2$$

where m = mass in kilograms (kg), and v is the speed in m/s

The unit of kinetic energy is the joule (J)

If an object's speed is doubled, then its kinetic energy is quadrupled. This is because the kinetic energy of an object is proportional to its $(\text{speed})^2$. Similarly, tripling a car's speed will increase its kinetic energy ninefold! Naturally this has very dangerous implications for cars travelling at high speeds.

Gravitational Potential Energy PE

Gravitational potential energy PE is energy that is "stored" due to its position in a gravitational field.

$$PE = mgh$$

where m = mass in kilograms (kg), g = gravitational acceleration (m/s^2),
and h = height above a reference point (m).

Work W and Energy E

A body that has energy may transfer some, or all, of its energy to another body. The total amount of energy remains constant (conserved) even if it has been transformed to another type. The amount of energy transformed (ΔE) is called work W. The body losing energy does work, the body gaining energy has work done on it.

Work is given by the force multiplied by the displacement through which the force acts, or:

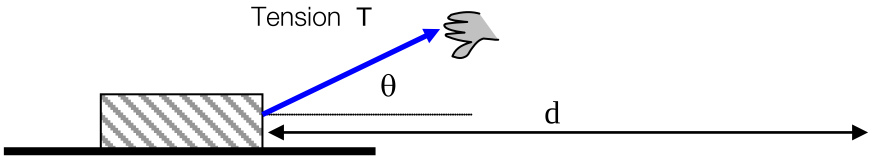
$$\text{Work} = \text{Change in Energy} = \text{Force} \times \text{displacement}$$

$$W = \Delta E = F \times d$$

where F = force (N), d = displacement (m)

Note: Work is a scalar quantity. The unit of work is the Joule (J)

Consider a force that acts at an angle. For instance, when a block is being pulled with tension T by a string whose angle of inclination is θ to the horizontal.



If the block moves a displacement d metres, then:

$$W = \Delta E = F \times d = T\cos\theta \times d$$

where $F = T\cos\theta$, the component of T in the horizontal direction.

Observe how the tension has been resolved in the direction of travel, ie, in the horizontal direction $T\cos\theta$.

Note: if the force is perpendicular to the displacement no work is done.
The reason: $W = F\cos\theta \times d = F\cos 90 \times d = 0$, where θ (90°) is the angle between the force and the displacement through which the force acts. The implication of this is that if the force acts at an angle to the direction of motion of an object, then the force is less effective compared with if the same force was acting parallel to the object's motion.

If an object is acted on by a force, then the work done on it is equal to the change in kinetic energy of the object:

$$W = \Delta E = \tfrac{1}{2} m v_f^{\,2} - \tfrac{1}{2} m v_i^{\,2}$$

where "v_i" is the initial speed and "v_f" is the final speed

Force-Distance graphs

If the force is non-uniform then the formula $W = F \times d$ does not apply. Instead, we present the information as a force-distance graph and calculate the area beneath the graph to find the work done by the force.

Recall the other "area under a graph", which is the force-time graph that gives the impulse of a force. Again, these graphs are only useful if the force is not uniform.

Power P

Power P is the rate at which work is done

$$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{W}{t} = \frac{F \times d}{t} = F \times v$$

where v is the speed d/t. The unit of power is the Watt (W).

Note: the formula $P = F \times v$ can only be used if an object is travelling at a constant speed.

Conservation of Energy

A fundamental principle of nature is that energy is conserved, meaning that while it may change from one form to another the total amount is always the same. Similarly energy may be transferred from one type (eg, potential energy) to another (eg, kinetic energy) but again the overall amount stays constant.

$$\Delta\ PE = \Delta\ KE$$

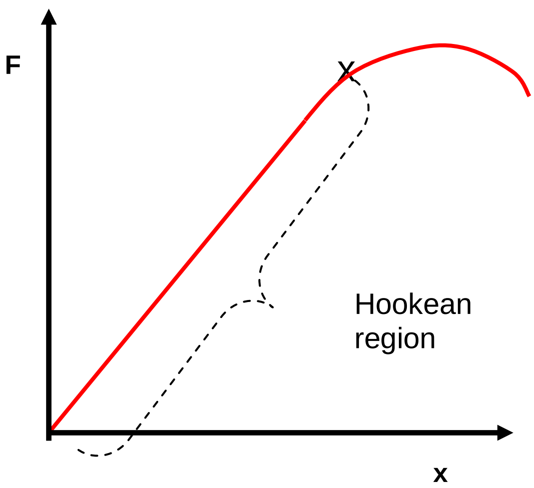
For instance, dropping a stationary ball of mass m from a height h involves a change in energy from potential to kinetic, or

$$\Delta(mgh) = \Delta(\tfrac{1}{2}\ mv^2)$$

Δ means "change in"

Question 2 of the exercise at the end of this worksheet is an illustration of conservation of energy.

Hooke's Law and Elastic Potential Energy



Hooke's Law states that the force F applied to a spring or similar object is proportional to the spring's extension or compression.That is,

$$F = -k\ x$$

where k = force (or spring) constant (N/m)
x = extension or compression

The straight line (the "Hookean" region) passing through the origin shown in the graph at left is given by the slope of the F-x graph. It gives a measure of the stiffness of the material: the steeper the slope the stiffer the material.

Note: The negative sign in the formula above tells us that the "restoring force", the ability of a spring to pull back (or extend) to its original length, is in the opposite direction to the applied force.

Elastic limit

Objects may obey Hooke's Law initially (the straight line section of the graph), but will reach a point where they becomes non linear (no longer proportional). This point is called the elastic limit of the spring and is shown by the X on the graph above. The object may start to stiffen, it could weaken (like chewing gum), or it may eventually break. Stretching an object beyond the elastic limit produces permanent changes to the structure of the material such that even when the force is released it doesn't behave the same, and will not return to its original length.

Elastic (Strain) Potential Energy U_s

The elastic potential energy Us, also known as the strain energy, is the energy stored in an elastic material, and is given by:

$$U_s = \frac{1}{2} kx^2$$

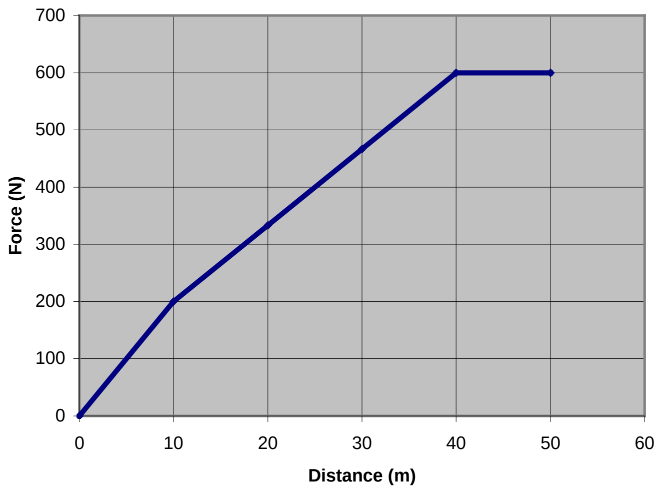
where k = spring constant, x = extension (or compression) of the spring.

Force-Extension (or Compression) Graph

Just as the area under a force-displacement graph gives us the work done by a force (see above), so the area under a force-extension (or compression) graph allows us to calculate the work done by a spring.

Exercise

1. The graph below shows how the total resistance forces acting on a cyclist and her bicycle vary with distance at the start of a race. The cyclist applies a constant force over the first 50m of the race and travels at a constant velocity after travelling 40m. The cyclist and bicycle have a combined mass of 80kg.

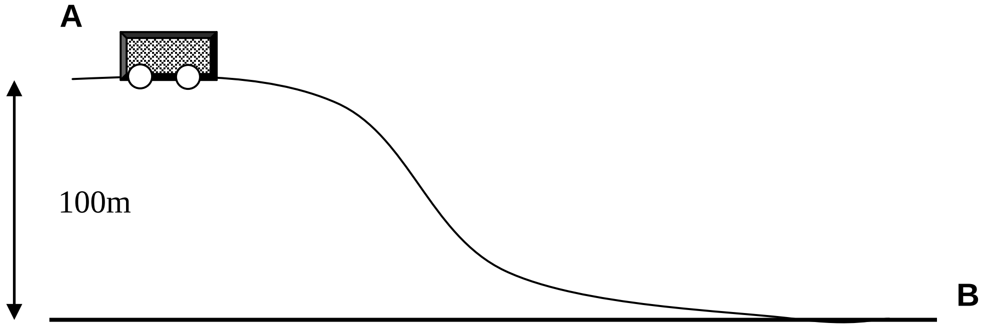


(a) How much work does the cyclist do against the resistance force over the first 40m of the race?

(b) Calculate the power developed by the cyclist during the first 40m of the race if she took 10 seconds to cover this distance.

(c) What was the magnitude of the constant force applied by the cyclist over the first 50m of the race?

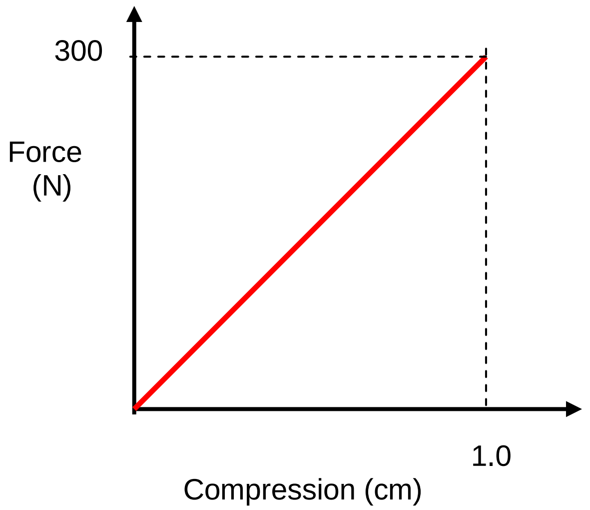
2. A car is shown at the top of an extremely steep hill, of height 100m, as shown below. The mass of the car and driver is 1200kg. The car is at rest at point A.



(a) Assuming frictional forces are ignored, calculate the speed that the car has at point B if it is allowed to roll down the hill without the driver applying the brakes.

(b) Assuming the car applies the brakes and does 1.0×10^6 J of work against friction as it rolls down the hill, what is the speed of the car at B now?

3. The graph below shows how the force applied by a pinball spring plunger changes as it is compressed during a pinball game.



The plunger is compressed 1cm and then released. If the pinball has a mass of 50 grams, what is its speed at the instant it leaves the plunger?

Answers

1	(a) 1.3×10^4J	(b) 1.3×10^3W	(c) 600N
2	(a) 45 m/s	(b) 18 m/s	
3	10m/s		