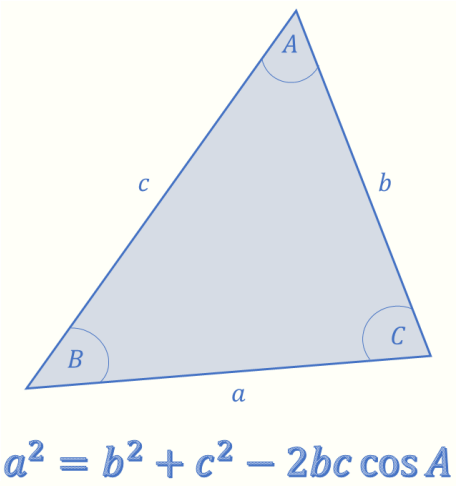
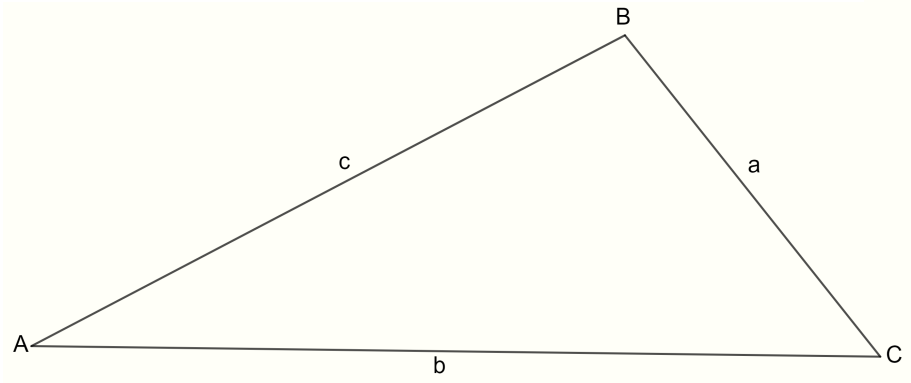
T4: Cosine Rule



Pythagoras's Theorem may be used to find the third side in any right angled triangle. The Cosine Rule can be used to solve non-right triangles.

The Cosine Rule

Consider the triangle below:



The angles A, B, C are the angles at the vertices A, B, C respectively. The sides a, b, and c are opposite angles A, B, C respectively.

The Cosine Rule states:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$

Note that the side on the left hand side of the equation is opposite the angle listed at the end of the equation:

$$\underline{\boldsymbol{a^2}} = b^2 + c^2 - 2bc\cos\underline{\boldsymbol{A}}.$$

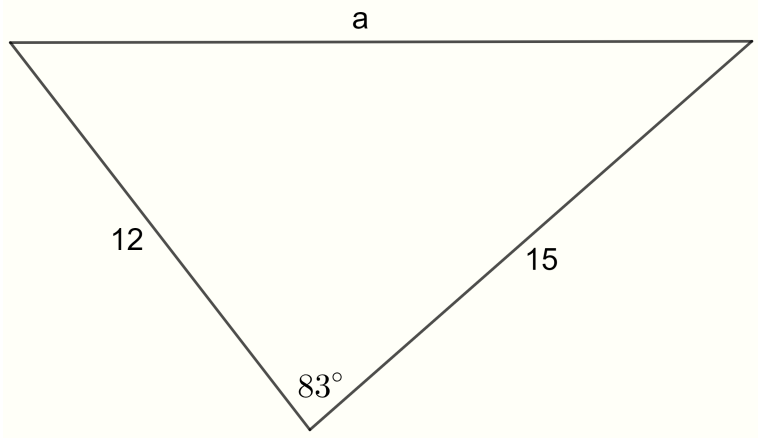
Use the Cosine Rule when you are given:

1. two sides and the angle between them, or

2. all three sides of the triangle.

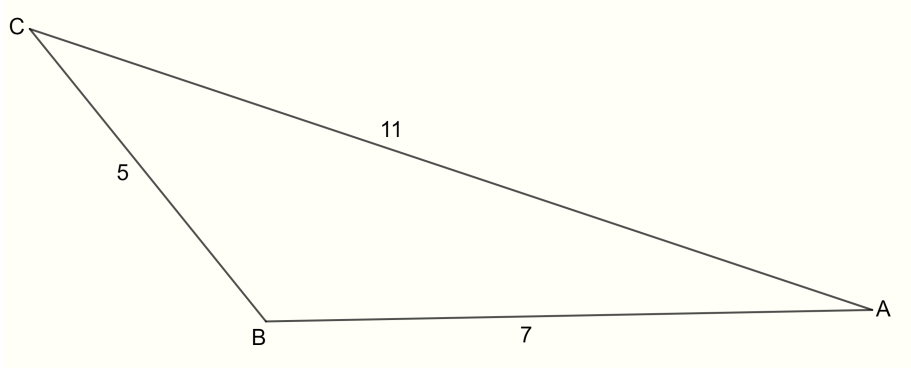
Examples

1. Find the value of a in this triangle



$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
a^2 &= 12^2 + 15^2 - 2 \times 12 \times 15 \times \cos 83^\circ \\
a^2 &= 144 + 225 - 360 \times \cos 83^\circ \\
a^2 &= 369 - 43.87 \\
a^2 &= 325.13 \\
a &= 18.03
\end{aligned}
$$

2. Find the size of angle B in this triangle:

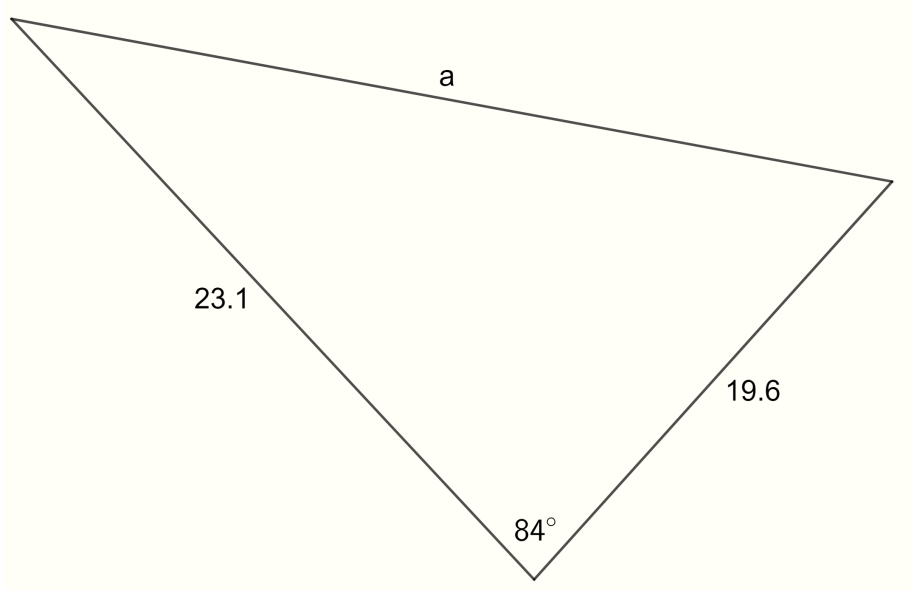


$$
\begin{aligned}
b^2 &= a^2 + c^2 - 2ac\ \cos B \\
11^2 &= 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos B \\
121 &= 25 + 49 - 70 \times \cos B \\
121 - 25 - 49 &= -70 \times \cos B \\
47 &= -70 \times \cos B \\
\frac{47}{-70} &= \cos B \\
B &= \cos^{-1}\left(-\frac{47}{70}\right) \\
B &= 132^\circ 11'
\end{aligned}
$$

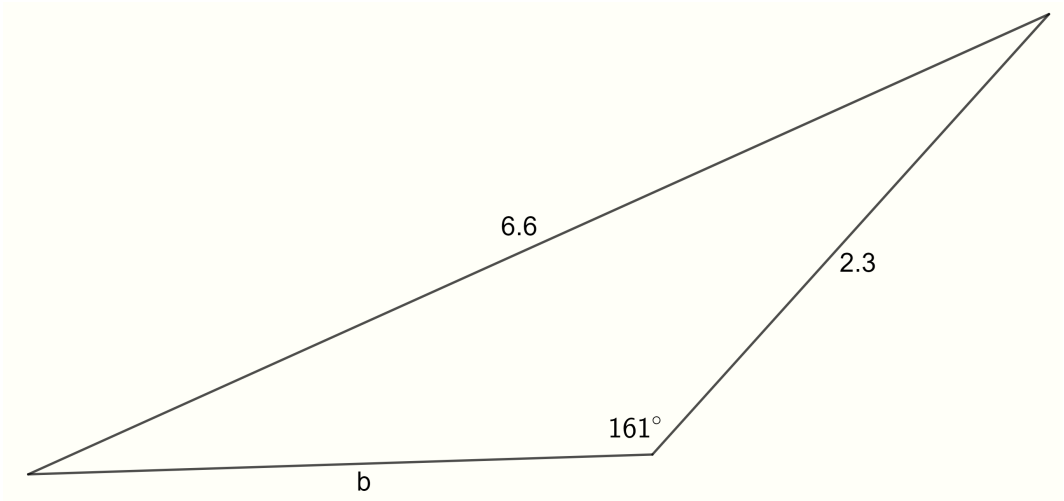
Exercise

1. Use the sine OR cosine rule to find the pro-numeral shown:

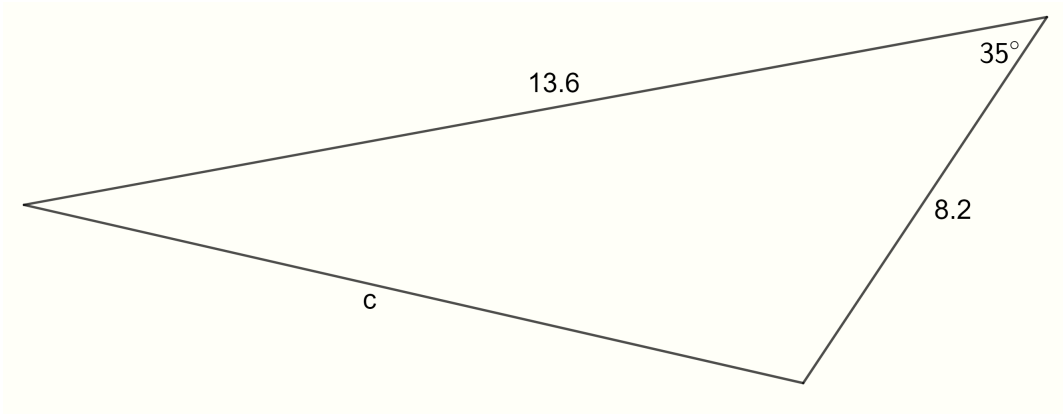
a)



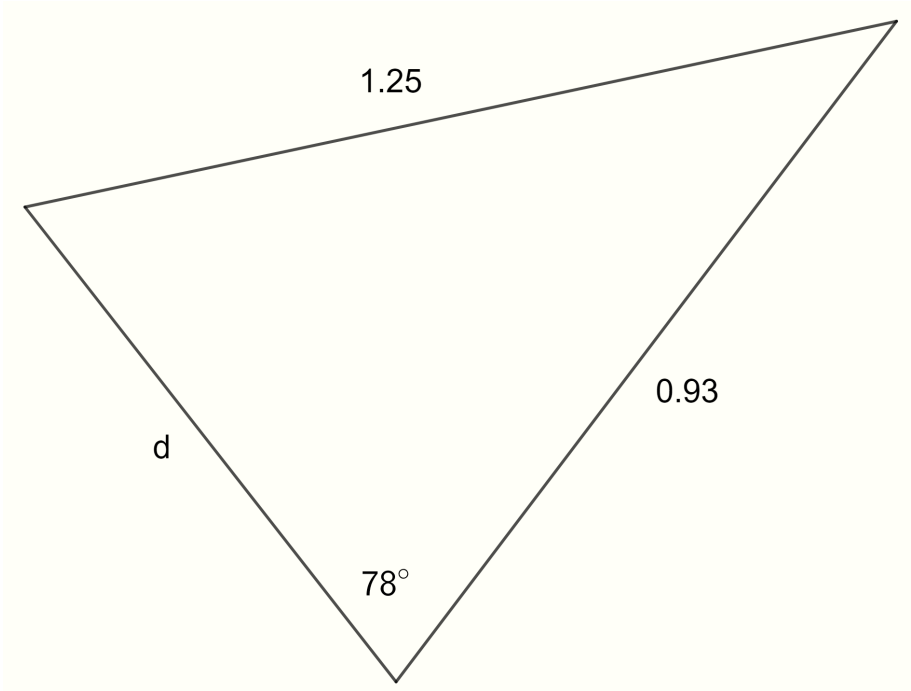
b)



c)

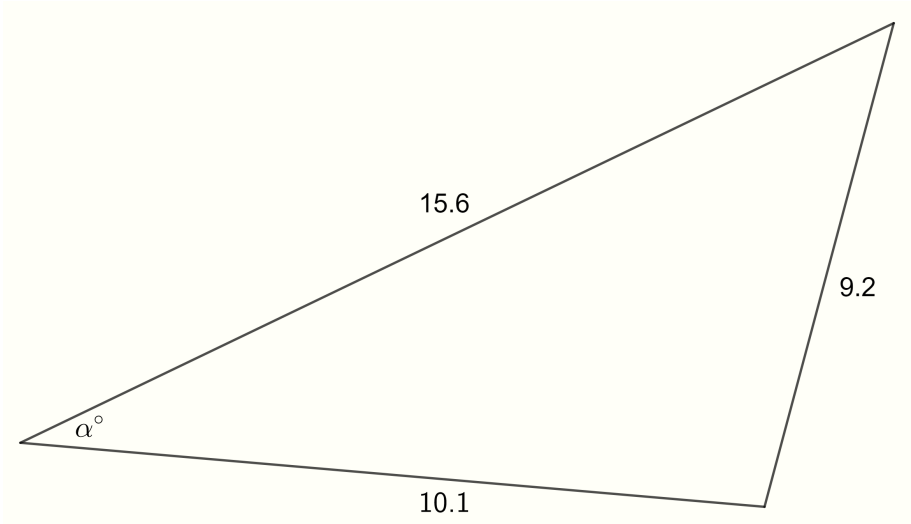


d)

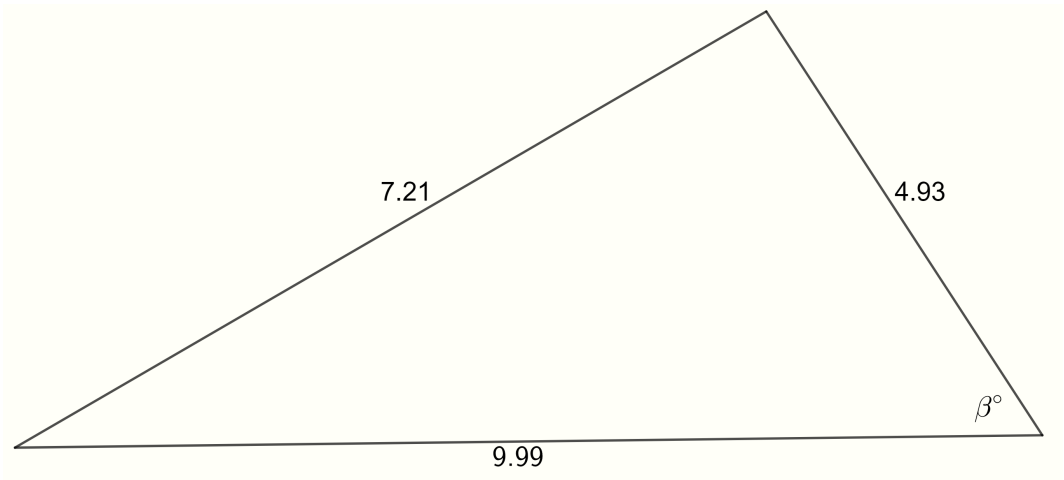


2. Find the magnitude of the labeled, unknown angle

a)



b)



Answers

1 a) 28.7 b) 4.38 c) 8.33 d) 1.05
2 a) $\alpha = 34.2^\circ$ b) $\beta = 42.9^\circ$.