

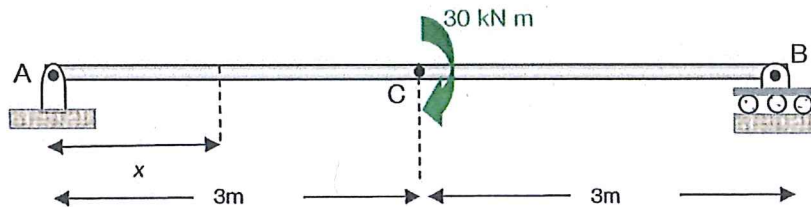
WORKED SOLUTIONS

ENST2.4: SHEAR AND BENDING MOMENT DIAGRAMS

Question 1

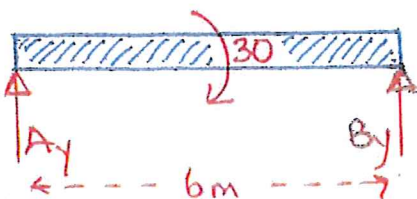
Determine the shear and moment as a function of x , where $0 \leq x < 3\text{m}$ and $3 < x \leq 6\text{m}$, and then draw the shear and moment diagrams.

(Hibbeler, R.C., 2010 12th Ed., *Statics*, Pearson)



Worked Solution 1

Support Reactions

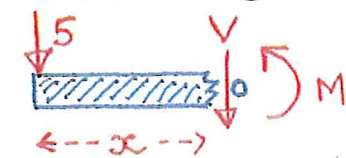


$$+\circlearrowleft \sum M_A = 0 : -30 + 6B_y = 0 \Rightarrow B_y = 5\text{ kN}$$

$$+\uparrow \sum F_y = 0 : A_y + 5 = 0 \Rightarrow A_y = -5\text{ kN}$$

Shear & Moment Reactions

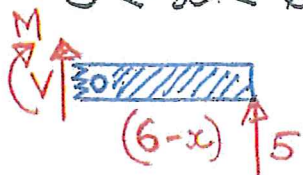
$$0 \leq x < 3$$



$$+\uparrow \sum F_y = 0 : -5 - V = 0 \Rightarrow V = -5\text{ kN}$$

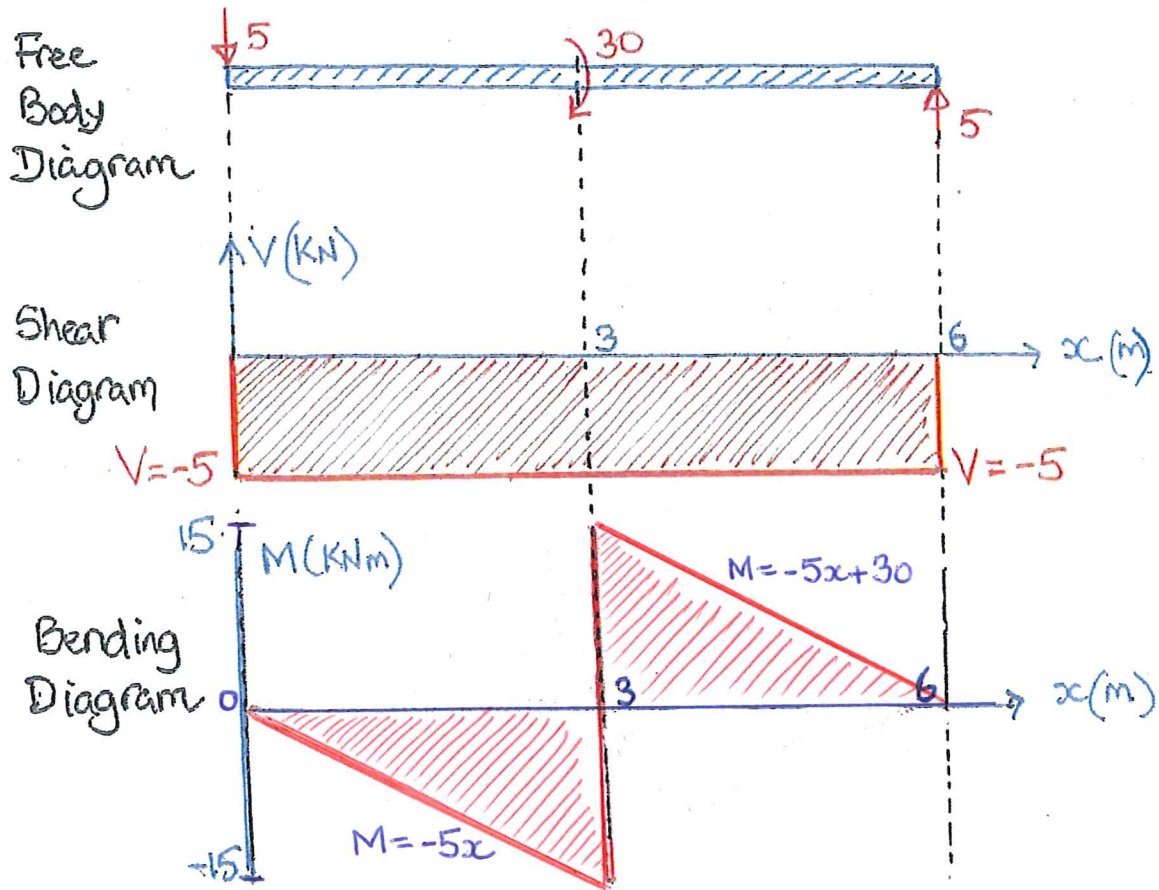
$$+\circlearrowleft \sum M_o = 0 : M + 5x = 0 \Rightarrow M = -5x$$

$$3 \leq x \leq 6$$



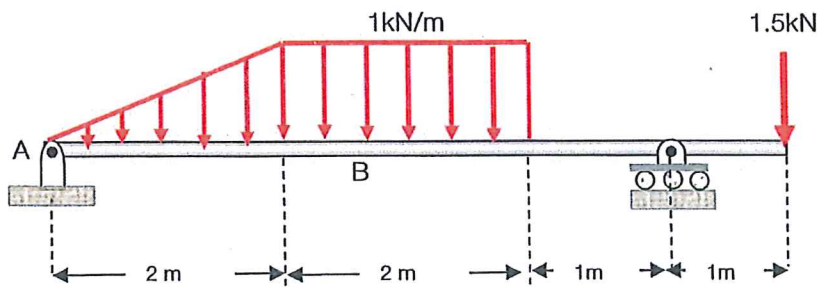
$$+\uparrow \sum F_y = 0 : V + 5 = 0 \Rightarrow V = -5\text{ kN}$$

$$+\circlearrowleft \sum M_o = 0 : 5(6-x) - M = 0 \Rightarrow M = -5x + 30$$

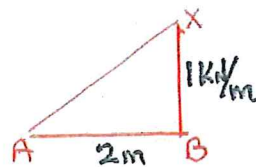
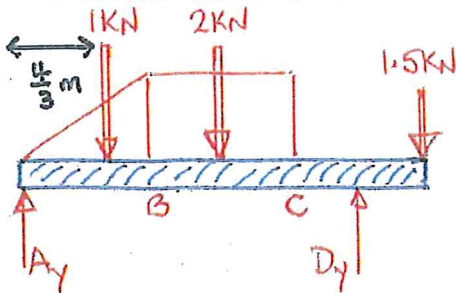


Question 2

Draw the shear and moment diagrams for the loaded beam below.
(Meriam, J.L. & Kraige, 2008 6th Ed., Statics, Wiley)

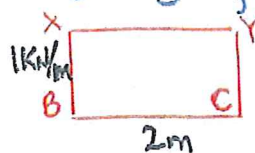


Worked Solution 2



$$\text{Load} = \text{Area } \triangle ABX \\ = \frac{1}{2} \times 2 \times 1 = 1 \text{ kN}$$

This load acts through centroid of \triangle ,
or $\frac{2}{3}$ of $2\text{m} = \frac{4}{3}\text{m}$.



$$\text{Load} = \text{Area } \square BCX \\ = 2 \times 1 = 2 \text{ kN, which acts through centroid of } \square$$

Support Reactions

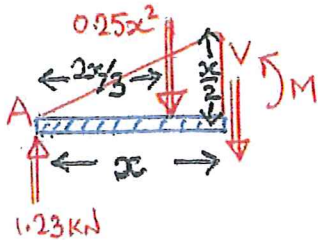
$$+\circlearrowleft \sum M_A = 0 : (-1 \times \frac{4}{3}) - (2 \times 3) - (1.5 \times 6) + (D_y \times 5) = 0$$

$$\Rightarrow D_y = 3.27 \text{ kN}$$

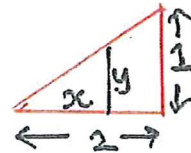
$$+\uparrow \sum F_y = 0 : A_y + 3.27 - 1 - 2 - 1.5 = 0 \Rightarrow A_y = 1.23 \text{ kN}$$

Shear & Moment Reactions

$$0 \leq x < 2$$



Using similar Δ 's :



$$y = \frac{x}{2} \text{ kN/m}$$

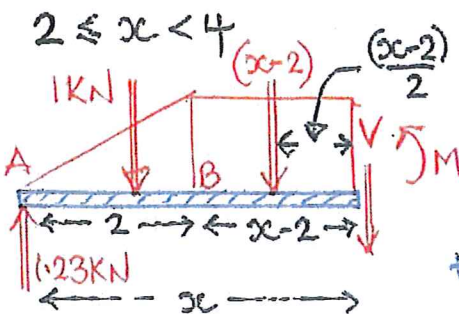
$$\text{Area of } \Delta = \frac{1}{2} \times x \times \frac{x}{2} = 0.25x^2$$

which acts through centroid of Δ : $\frac{2}{3}$ of $x = \frac{2x}{3}$

$$+\uparrow \sum F_y = 0 : 1.23 - 0.25x^2 - V = 0 \Rightarrow V = 1.23 - 0.25x^2$$

$$+\circlearrowleft \sum M = 0 : M + (0.25x^2 \times \frac{2x}{3}) - 1.23x = 0 \Rightarrow M = 1.23x - 0.083x^3$$

$$2 \leq x < 4$$



$$\text{Area of } \square = (x-2) \times 1 = (x-2)$$

which acts through centroid of $\square = \frac{(x-2)}{2}$

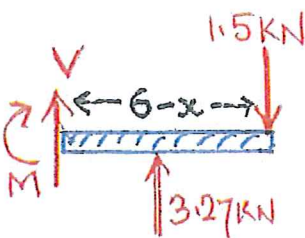
$$+\uparrow \sum F_y = 0 : 1.23 - 1 - (x-2) - V = 0$$

$$\Rightarrow V = 2.23 - x$$

$$+\circlearrowleft \sum M = 0 : M + \left\{ (x-2) \left(\frac{x-2}{2} \right) \right\} + \left\{ 1 \times \left(x - \frac{2}{3} \cdot 2 \right) \right\} - (1.23 \times x) = 0$$

$$\Rightarrow M = -0.67 + 2.23x - 0.5x^2$$

$$4 \leq x \leq 6$$

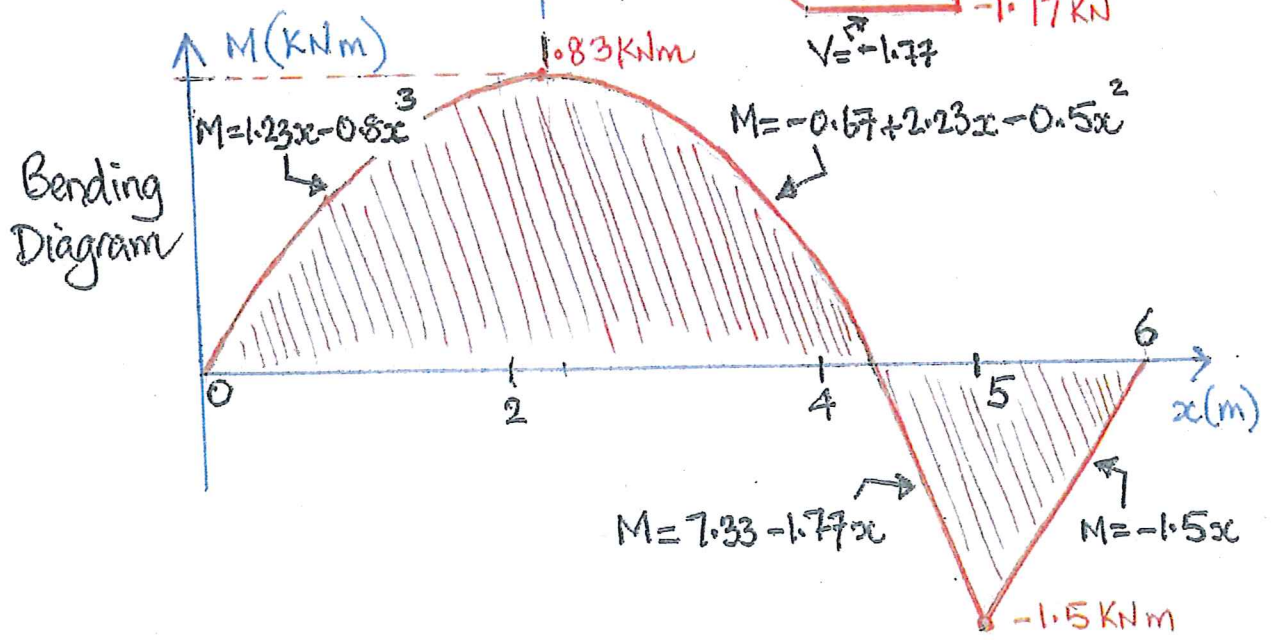
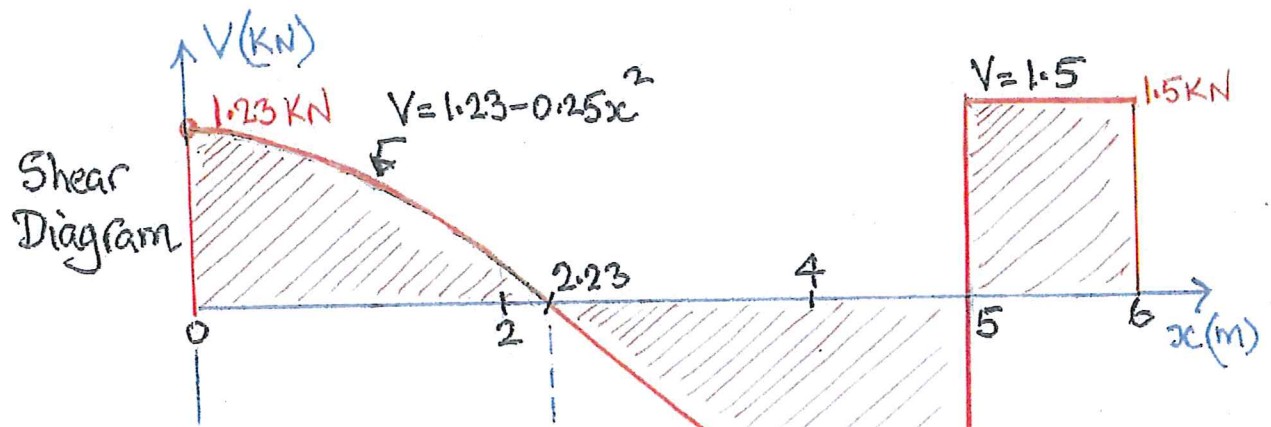
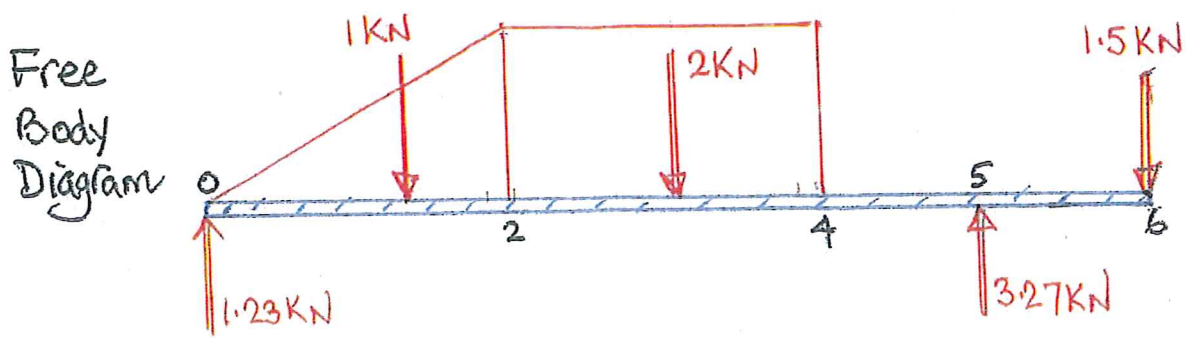


$$+\uparrow \sum F_y = 0 : V + 3.27 - 1.5 = 0$$

$$\Rightarrow V = -1.77 \text{ kN}$$

$$+\circlearrowleft \sum M = 0 : -M - 1.5(6-x) + 3.27 \times \frac{6-x}{2} = 0$$

$$\Rightarrow M = 7.33 - 1.77x$$



Note:

- Maximum moment occurs at $x = 2.23\text{m}$ where shear curve crosses the X-axis, $M_{\text{MAX}} = 1.83\text{ kNm}$
- Change in moment ΔM up to any section equals the area under the shear diagram up to that section.