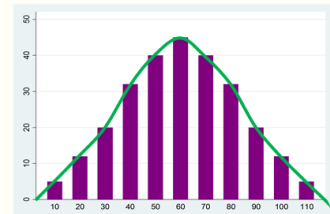
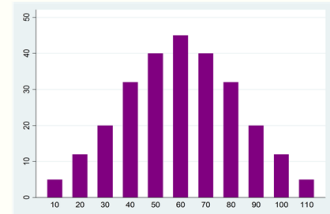


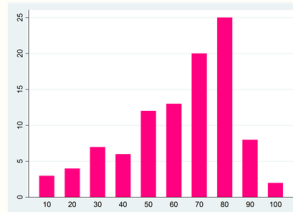
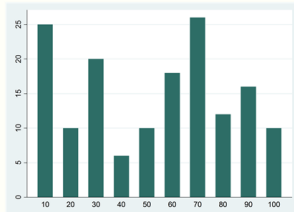
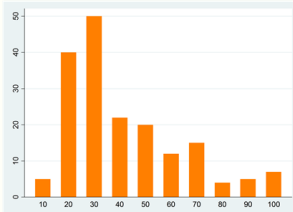
S9 Normal Distributions

Introduction

This module introduces the normal distribution. Data that is normally distributed is characterized by a bell shaped curve when displayed graphically.



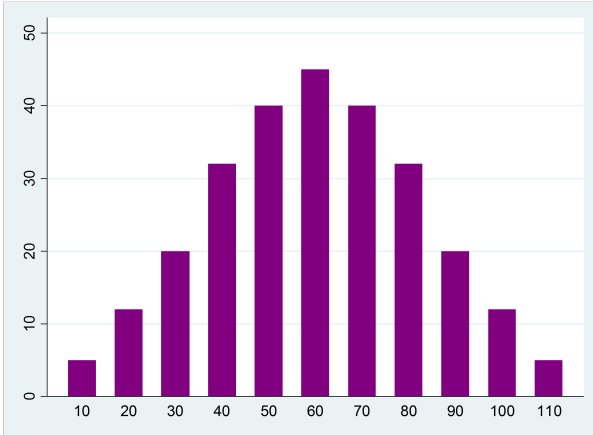
Graphical representations of data may look quite different as shown below:



But many things that can be measured, such as

- heights of people
- blood pressure
- errors in measurement
- scores on a test

follow a bell shaped curve like the figure below.



Such data is said to be normally distributed.

Properties of a Normal Distribution

1. Symmetry about the mean
2. Mean = median = mode
3. 50% of values greater than the mean and 50% less than the mean
4. 68% of values fall within one standard deviation either side of the mean (approximately)
5. 95% of values fall within two standard deviations either side of the mean (approximately)
6. 99.7% of values fall within three standard deviation either side of the mean (approximately)

Note: Even though most of the data will fall within three standard deviations of the mean there is in theory, no upper or lower bound to a normal distribution. We are just less and less likely to find values beyond these points.

Example

If scores on an IQ test are normally distributed with mean = 100 and standard deviation = 10, what percentage of people would we expect to

- (a) score between 90 and 110?
- (b) score less than 80?

Solution:

(a) Because $90 = 100 - 10$ and $110 = 100 + 10$ are both one standard deviation from the mean, 68% of people would be expected to score between 90 and 110.

(b) $80 = 100 - 2 \times 10$ is two standard deviations below the mean. We know that 95% of scores fall between 80 and 120 so 5% must fall outside this range. Half of these, 2.5%, will be below 80. Therefore we would expect that 2.5% of people to have IQ scores less than 80.

Exercises

1. Scores on a general achievement test are normally distributed with a mean of 80 and a standard deviation of 15. Adam scored 95. What proportion of students had a higher score than Adam?

Answer: 16%

2. The actual weights of cereal boxes that are supposed to contain 500g are normally distributed with mean of 510g and a standard deviation of 5g. What proportion of boxes are under-filled?

Answer: 2.5%

3. In a maths class the bottom 16% of students are given an F grade. If the class mean is 63 and the standard deviation is 18 what score must a student get to pass?

Answer: 45

4. If newborn birth weights in a certain hospital are normally distributed with a mean of 3200g and a standard deviation of 400g

(a) what percentage of babies weigh more than 3200g?

(b) what percentage of babies weigh between 2400g and 4000g?

(c) what percentage of babies weigh less than 3600g?

(d) if the 16% of babies with the lowest birth weights are placed in the special care nursery will a baby that weighs 2500g need special care?

Answer: (a) 50% (b) 95% (c) 84% (d) yes

5. 95% of people in a clinical study had systolic blood pressure readings between 116 and 144. If the blood pressure measurements follow a normal distribution what is the mean and standard deviation of the blood pressures for this group.

Answer: $\mu = 130, \sigma = 7$

6. A class of ten students get the following marks in a test: 13, 23, 41, 55, 66, 78, 49, 33, 35, 67. If anyone who scored less than one standard deviation below the mean fails how many students will fail?

Answer: 2