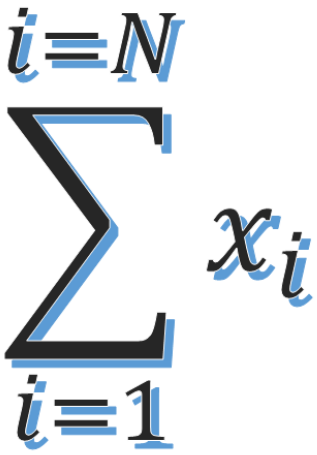
S1: *Summation Notation*

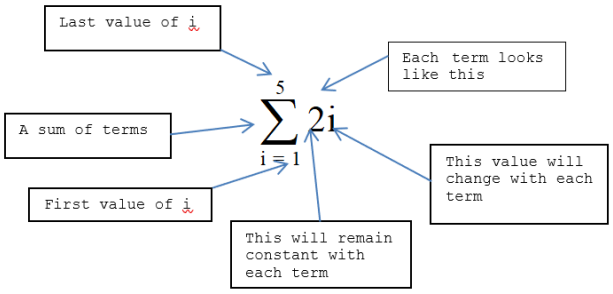


Summation notation or sigma notation is a shorthand method of writing the sum or addition of a string of similar terms. This module explains the use of this notation.

The Basic Idea

We use the Greek symbol sigma Σ to denote summation. Σ is called the summation sign.

A typical element of the sequence which is being summed appears to the right of the summation sign as shown in the figure below:



This is written as $\sum_{i=1}^{5} 2i$.

To expand and work out it's value, we replace i by its starting value (below the sigma symbol) and obtain each successive term by adding 1 to the previous value until we reach the final value of i (above the sigma symbol) and then we evaluate.

For the above sequence:

$$\begin{aligned}\sum_{i=1}^{5} 2i &= 2\times 1 + 2\times 2 + 2\times 3 + 2\times 4 + 2\times 5 = 30\\ &= 2+4+6+8+10\\ &= 30.\end{aligned}$$

Example

Expand and evaluate $\sum_{i=0}^{3}(i^2-3)$

Note that only the i value changes in each term.

$$\begin{aligned}\sum_{i=0}^{3}(i^2-3) &= \underbrace{(0^2-3)}_{i=0}+\underbrace{(1^2-3)}_{i=1}+\underbrace{(2^2-3)}_{i=2}+\underbrace{(3^2-3)}_{i=3}\\ &= (-3)+(-2)+1+6\\ &= 2.\end{aligned}$$

Data Sets

Subscripted variables are used to indicate values in a data set. x_1, x_2, x_3... refers to first value, second value, third value and so on. Formulae used to calculate summary measures of a data set make use of summation notation.

Example

Given the set of data $x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 5$ evaluate

a) $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ where n is the number of items in the data set

This formula calculates the mean or average of a set of data.

$$\begin{aligned}\overline{x} &= \frac{\sum_{i=1}^{4} x_i}{4}\\ &= \frac{x_1+x_2+x_3+x_4}{4}\\ &= \frac{1+2+4+5}{4}\\ &= 3\end{aligned}$$

b) $s^2 = \frac{\sum_{i=1}^{4}(x_i-\overline{x})^2}{n-1}$

This formula calculates the variance (a measure of spread around the mean) of a sample of data.

$$\begin{aligned}s^2 &= \frac{\sum_{i=1}^{4}(x_i-\overline{x})^2}{n-1}\\ &= \frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+(x_3-\overline{x})^2+(x_4-\overline{x})^2}{3}\\ &= \frac{(1-3)^2+(2-3)^2+(4-3)^2+(5-3)^2}{3}\\ &= \frac{4+1+1+4}{3}\\ &= \frac{10}{3}.\end{aligned}$$

Exercise 1

Find

(a) $\sum_{i=1}^{5} 3i$
(b) $\sum_{i=1}^{3}(5i-2)$
(c) $\sum_{i=1}^{3}(5i)-2$

Answers

1. a. 45 b. 24 c. 28

Exercise 2

Given $x_1 = -2$, $x_2 = 0$, $x_3 = 1$, $x_4 = 3$, $x_5 = 3$ evaluate

(a) $\sum_{i=1}^{5} 10x_i$
(b) $10\sum_{i=1}^{5} x_i$
(c) $\sum_{i=1}^{5} (x_i)^2$
(d) $\left(\sum_{i=1}^{5} x_i\right)^2$
(e) $\sum_{i=1}^{5} i \times (x_i)$
(f) $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

Answers

2. a. 50 b. 50 c. 23
d. 25 e. 28 f. 1