RMIT UNIVERSITY

WORKED SOLUTIONS

NM2.5: NUMERICAL METHODS: EULER'S METHOD

Question

Find, using Euler's method, the approximate solution of $y' = x + 2y$ on the interval $[0, 0.5]$ with a step size of 0.1 given $y(0) = 1$.

Worked Solution

Euler's Formula is $y_{n+1} = y_n + hf(x_n, y_n)$ $n = 0, 1, 2, \ldots$

which can be written as $y_{n+1} = y_n + h(y')_n$ where $(y')_n = f(x_n, y_n)$

Also $x_{n+1} = x_n + h$ where h is the step size

Since $y' = x + 2y$ we have $(y')_n = x_n + 2y_n$

$n = 0$: $x_0 = 0$, $y_0 = 1$, $h = 0.1$, $(y')_0 = ?$ $y_1 = ?$

(since $y(0) = 1$)

Sub into $(y')_n = x_n + 2y_n$ we get $(y')_0 = x_0 + 2y_0$

and so $(y')_0 = 0 + 2(1) = 2$

Sub $(y')_0 = 2$ into $y_{n+1} = y_n + h(y')_n \Rightarrow y_1 = y_0 + h(y')_0$

we get $y_1 = 1 + 0.1(2) = 1.2$

$n=1$: $x_1 = x_0 + 0.1 = 0 + 0.1 = 0.1$, $y_1 = 1.2$, $h = 0.1$, $(y')_1 = ?$, $y_2 = ?$

sub into $(y')_1 = x_1 + 2y_1$ we get $(y')_1 = 0.1 + 2(1.2) = 2.5$

sub $(y')_1 = 2.5$ into $y_2 = y_1 + h(y')_1$ we get $y_2 = 1.2 + 0.1(2.5)$

$y_2 = 1.45$

Continue in this way $(n = 2, 3, 4, 5)$ until the interval $[0, 0.5]$ is complete.

Complete a table similar to this one.

n	x_n	y_n	$(y')_n$
0	0	1	2
1	0.1	1.2	2.5
2	0.2	1.45	3.1
3	0.3	1.76	3.82
4	0.4	2.142	4.684
5	0.5	2.6104	5.6208

The approximate solution $y(0.5) = 2.6104$

Solving this DE exactly we get $y = \frac{5}{4}e^{2x} + \frac{1}{4}(-2x-1)$

$\Rightarrow y(0.5) = 2.8978$

Better accuracy can be achieved using smaller values of h.