RMIT UNIVERSITY

WORKED SOLUTIONS

ENDY2.2 MOTION: NON-CONSTANT ACCELERATION

Question 1

(Hibbeler, R.C, 2010, Engineering Mechanics: Dynamics 12th Ed. Pearson)

The position of a particle is given by $s = t^3 - 3t^2$ m, where t is in seconds. Calculate the time(s) when (a) the velocity of the particle is zero, (b) the acceleration is zero, and (c) calculate the total distance travelled by the particle after 3 seconds.

Worked Solution 1

(a) $s = t^3 - 3t^2 \Rightarrow v = \frac{ds}{dt} = 3t^2 - 6t$

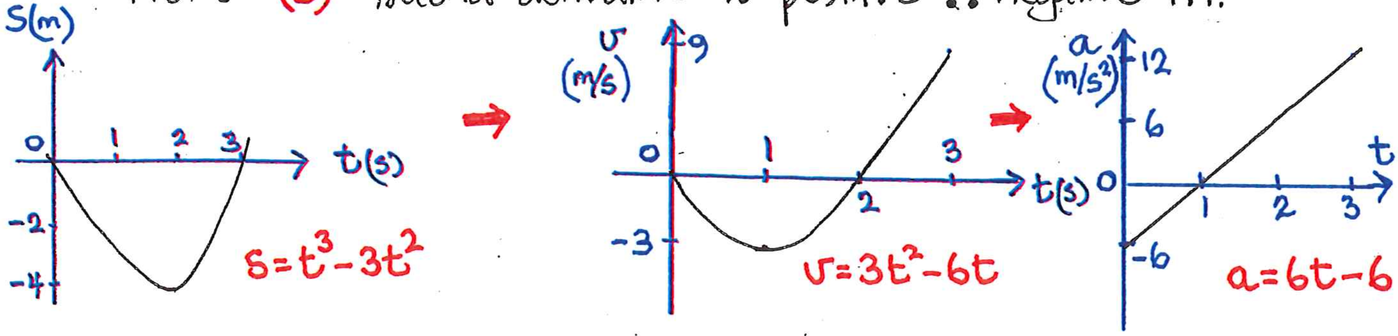
when $v = 0$, $3t^2 - 6t = 0 \Rightarrow 3t(t-2) = 0$
$\Rightarrow \underline{t = 0, 2\,s}$

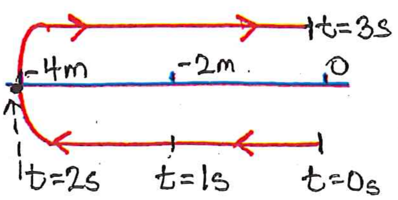
(b) $v = 3t^2 - 6t \Rightarrow a = \frac{dv}{dt} = 6t - 6$

when $a = 0$, $6t - 6 = 0 \Rightarrow \underline{t = 1\,s}$

(c) $s = t^3 - 3t^2 = t^2(t-3)$ Sketch a graph to see what's happening.

when $s = 0$, $t = 0, 3\,s$ (s-intercepts on s-t graph)
From (a) $t = 2s$ is turning point $0 \le t \le 3$
From (b) second derivative is positive $\therefore$ negative T.P.





Path of Motion

Distance travelled
$0 \le t \le 2$ $d = 4m$ to 'left'
$2 < t \le 3$ $d = 4m$ to 'right'

Total Distance travelled $= \underline{8m}$

Question 2 (Hibbeler, R.C, 2010, Engineering Mechanics: Dynamics 12th Ed. Pearson)

A particle is moving along a straight line such that its acceleration is given by $a = (-2v)$ m/s². If $v = 20$ m/s when $s = 0$ and $t = 0$, calculate (a) the particle's velocity (b) the particle's position and (c) the particle's acceleration when $t = 2$ seconds.

Worked Solution 2

(a) $a = \frac{dv}{dt} = -2v \Rightarrow \frac{1}{2}\int \frac{dv}{v} = -\int dt$

when $t=0$, $v=20$: $\frac{1}{2}\int_{20}^{v} \frac{dv}{v} = -\int_{0}^{t} dt$

$\frac{1}{2}\left[\ln v\right]_{20}^{v} = -t \Rightarrow \frac{1}{2}(\ln v - \ln 20) = -t$

$\ln\left(\frac{v}{20}\right) = -2t \Rightarrow \frac{v}{20} = e^{-2t} \Rightarrow v = 20e^{-2t}$

when $t=2$, $v = 20e^{-4} = \underline{\underline{0.366 \text{ m/s}}}$

(b) $v = \frac{ds}{dt} = 20e^{-2t}$ (see above)

$\Rightarrow ds = 20e^{-2t}\,dt \Rightarrow \int ds = 20\int e^{-2t}\,dt$

when $t=0$, $s=0$: $\int_{0}^{s} ds = 20\int_{0}^{t} e^{-2t}\,dt$

$s = 20 \cdot -\frac{1}{2}\left[e^{-2t}\right]_{0}^{t} = -10\left(e^{-2t}-1\right)$

when $t=2$, $s = -10\left(e^{-4}-1\right) = \underline{\underline{9.817 \text{ m}}}$

(c) Since $a = -2v$ and $v = 20e^{-2t}$

$a = -2\left(20e^{-2t}\right) = -40e^{-2t}$

when $t=2$, $a = -40e^{-4} = \underline{\underline{-0.733 \text{ m/s}^2}}$