RMIT UNIVERSITY

M 1 Matrices: Introduction

$$\begin{bmatrix} 1 & 0 & -2 \\ 7 & 0.03 & 13 \\ 1.6 & 23 & 5 \end{bmatrix}$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

- A matrix is a rectangular array of elements.
- Matrices are usually denoted by upper case letters.
- The elements are usually written within brackets.
- The order or shape of the matrix is determined by the number of rows and columns of the matrix.
- The number of rows is always given first then the number of columns.

Example.

$$A = \begin{bmatrix} 1 & 2 & -9 \\ 2 & 5 & -3 \end{bmatrix}$$

A has 2 rows and 3 columns and is called a 2×3 matrix [1].

A matrix with m rows and n columns is called a matrix of order $m \times n$ [2].

[1] This is verbally stated as a 2 by 3 matrix

[2] This is verbally termed an "m by n matrix".

Square Matrix

A matrix with the same number of rows and columns is called a square matrix.

Example:

$$B = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}$$

B is a square 2×2 matrix

Unit Matrix

A unit (or identity) matrix is a square matrix with diagonal elements equal to one, and all other elements equal to zero. The unit matrix is usually denoted by I.

I_3 is a 3×3 unit matrix
Example:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Row Matrix

A matrix with one row is called a row matrix.
Example:

$$D = \begin{bmatrix} 2 & 1 & 0 & 4 \end{bmatrix}$$

is a 1×4 row matrix

Column Matrix

A matrix with one column is called a column matrix.
Example:

$$E = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$$

is a 3×1 column matrix

Zero Matrix

A zero matrix has all elements equal to zero. A zero matrix can be written as 0.
Example:

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

is a 2×2 zero matrix

Equal Matrices

For two matrices to be equal they must be the same shape and the corresponding elements must be equal.
If A equals B then

$$A = \begin{bmatrix} 2 & 5 & b \\ 5 & 3 & 1 \\ 2 & 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 & 7 \\ 5 & a & 1 \\ 2 & 0 & -2 \end{bmatrix}$$

If A and B are equal then $a = 3$ and $b = 7$.

Exercise:

1. Write down the order of the following matrices.
   a. $\begin{bmatrix} 7 & -5 & 0 \\ 6 & 2 & -1 \end{bmatrix}$
   b. $\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$
   c. $\begin{bmatrix} 2 \\ -4 \\ 1 \\ 1 \end{bmatrix}$
   d. $\begin{bmatrix} 1 & 1 \\ 3 & 0 \\ -2 & 3 \end{bmatrix}$
2. Which of the following matrices are equal?

$$A = \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$$

$$E = \begin{bmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 3 \end{bmatrix} \qquad G = \begin{bmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \\ 1 & 3 & 0 \end{bmatrix}$$

Answers

1. $a) 2 \times 3 \qquad b) 2 \times 2 \qquad c) 4 \times 1 \qquad d) 3 \times 2$ 2. A and D