

FG5 Hybrid Functions

Introduction

Functions which have different rules for each subset of the domain are called hybrid functions.

Sometimes they are referred to as piece-wise defined functions.

An example of a hybrid function is:

$$y = f(x) = \begin{cases} -x, & x \leq -1 \\ 1, & -1 < x < 1 \\ x, & x \geq 1. \end{cases}$$

Note that this hybrid function has three rules, each depending on the value of x in its domain. A hybrid function may have two or more rules.

Example 1

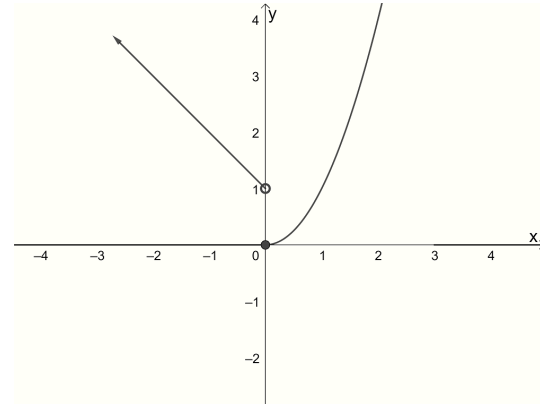
Graph the hybrid function

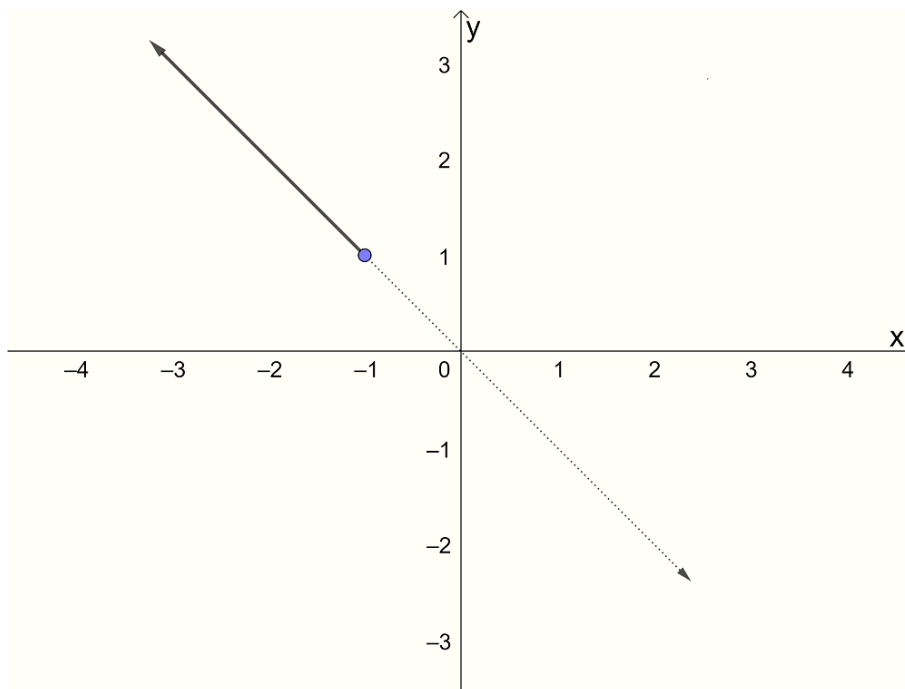
$$y = f(x) = \begin{cases} -x, & x \leq -1 \\ 1, & -1 < x < 1 \\ x, & x \geq 1. \end{cases}$$

Solution:

This is a hybrid function with three rules. We consider the graph of each of the rules, noting the restricted domains:

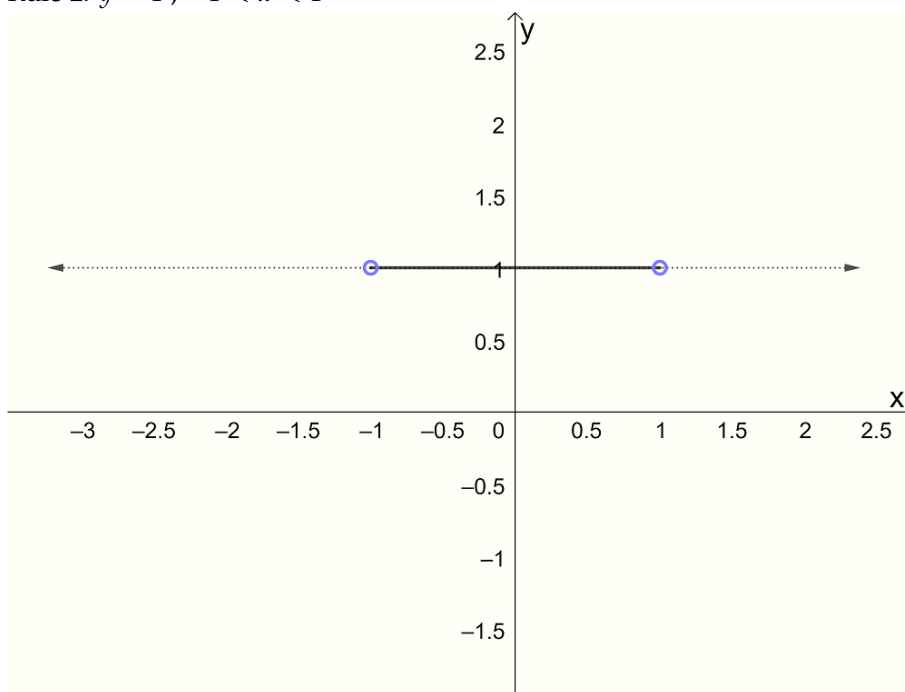
Rule 1. $y = -x, x \leq -1$





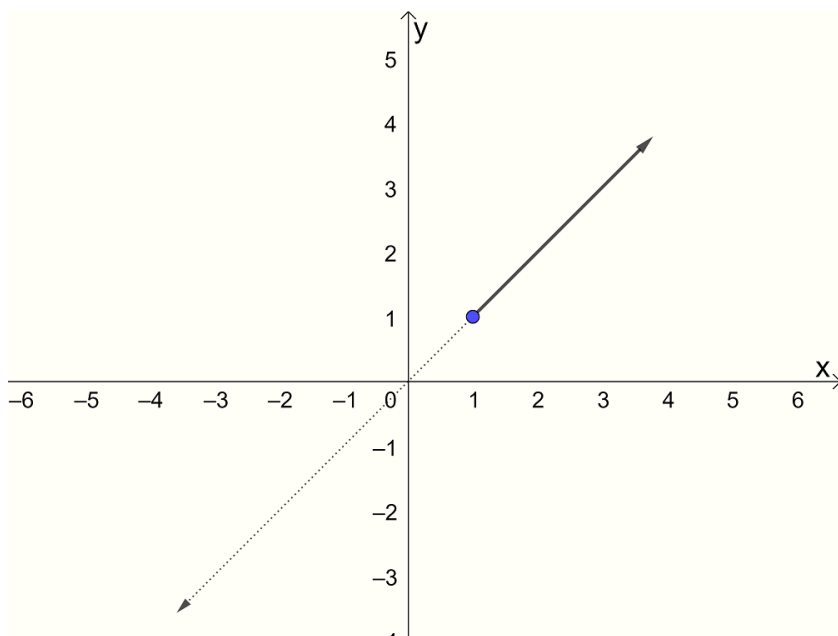
Note that the end point at $x = -1$ is marked with a filled in circle. This means -1 is in the domain of the function.

Rule 2. $y = 1, -1 < x < 1$



In this case, the open circles indicate that the points -1 and 1 are not included in the domain of the function.

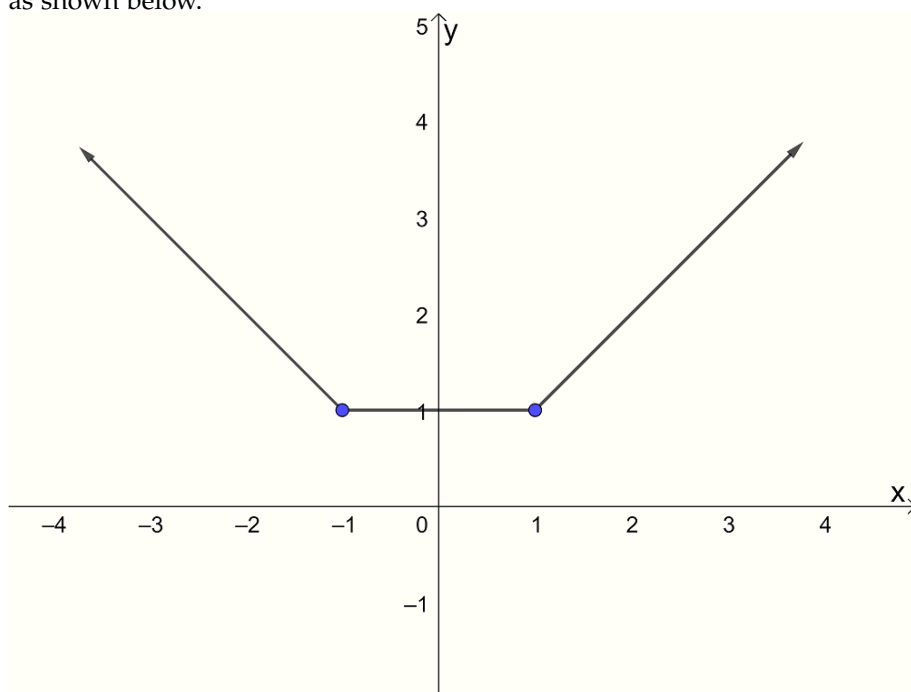
Rule 3. $y = x, x \geq 1$



The “graphical pieces” from rules 1 to 3 above can be put together to form the graph of the hybrid function

$$y = f(x) = \begin{cases} -x, & x \leq -1 \\ 1, & -1 < x < 1 \\ x, & x \geq 1 \end{cases}$$

as shown below.



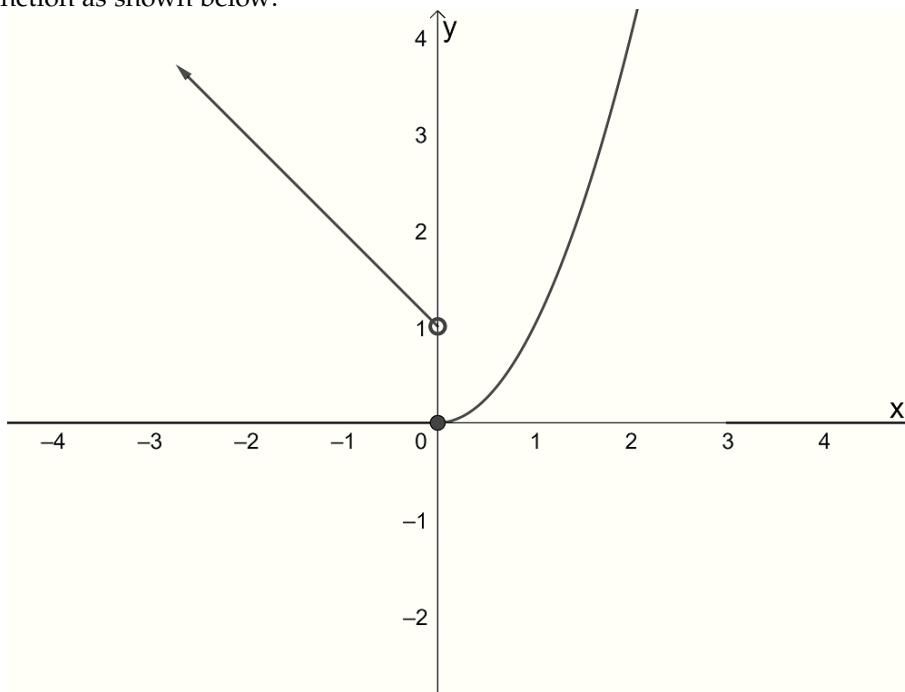
Example 2

Sketch the graph of

$$y = f(x) = \begin{cases} 1 - x, & x < 0 \\ x^2, & x \geq 0. \end{cases}$$

Solution:

This function has two rules. First rule is $f(x) = 1 - x$ for $x < 0$. The second rule is $f(x) = x^2$ for $x \geq 0$. Graphing each of these and assembling the “graphical pieces” gives the graph for the hybrid function as shown below:



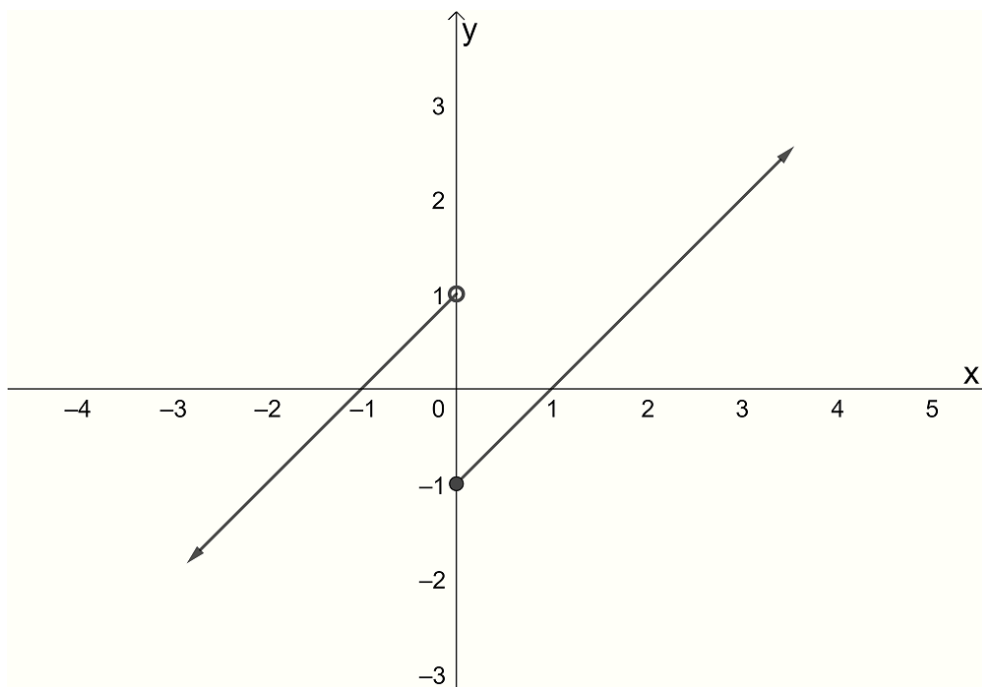
Note the open circle at $x = 0$ as this is not in the domain of the function $f(x) = 1 - x$. However, $x = 0$ is in the domain of $f(x) = x^2$ and so is shown with a filled dot.

Exercise

1. Draw a sketch graph of

$$f(x) = \begin{cases} x + 1, & x < 0 \\ x - 1, & x \geq 0. \end{cases}$$

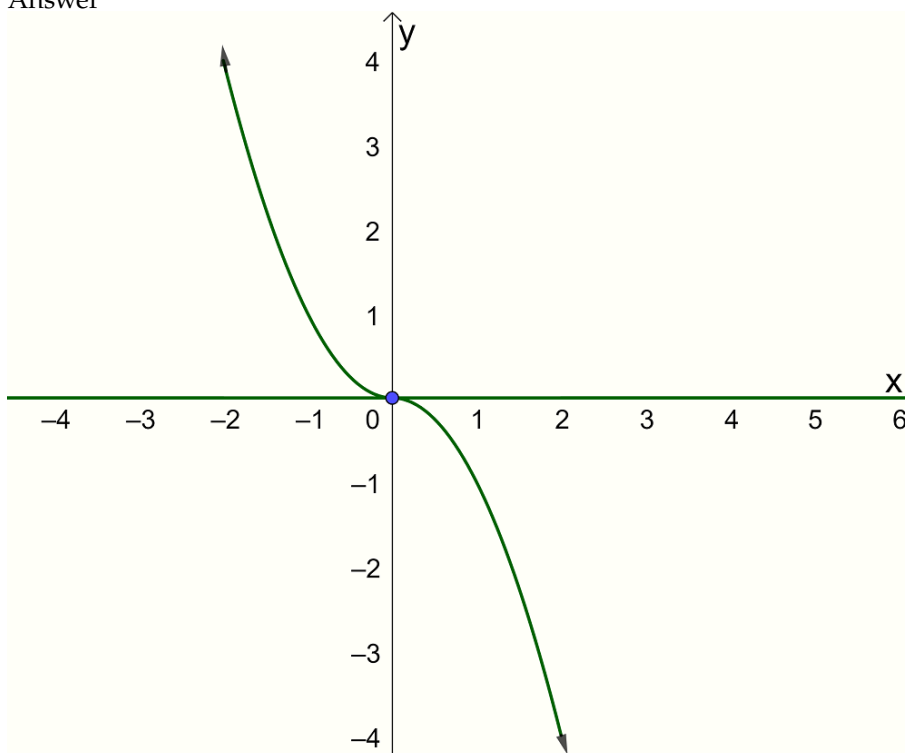
Answer



2. Draw a sketch graph of

$$f(x) = \begin{cases} x^2, & x < 0 \\ -x^2, & x \geq 0. \end{cases}$$

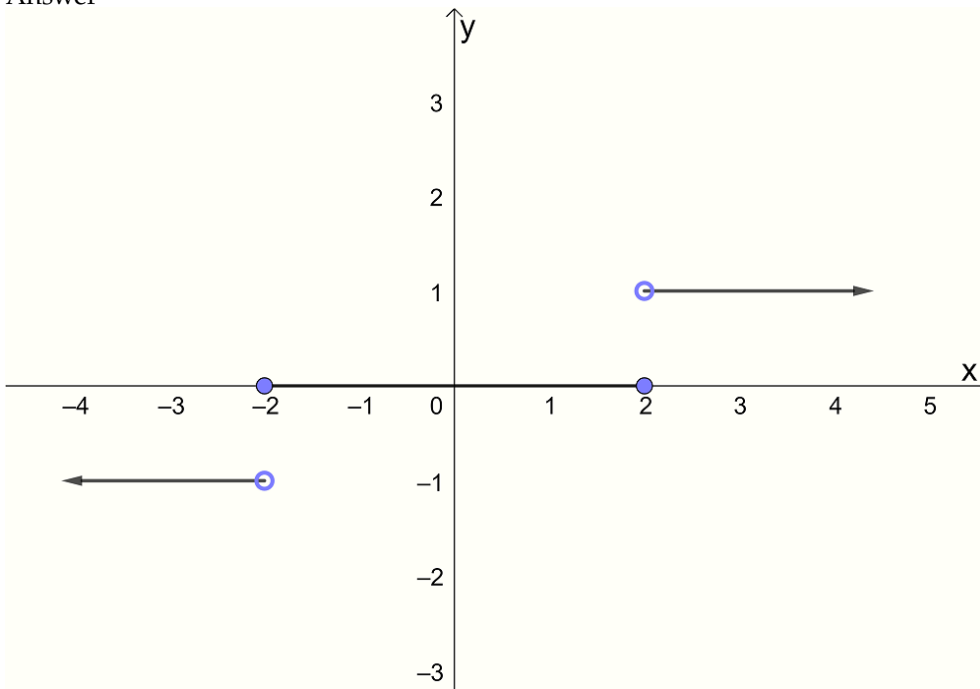
Answer



3. Draw a sketch graph of

$$f(x) = \begin{cases} -1, & x < -2 \\ 0, & -2 \leq x \leq 2 \\ 1, & x > 2. \end{cases}$$

Answer



4. Draw a sketch graph of

$$f(x) = \begin{cases} x + 2, & x < -1 \\ 1, & -1 \leq x \leq 1 \\ x, & x > 1. \end{cases}$$

Answer

