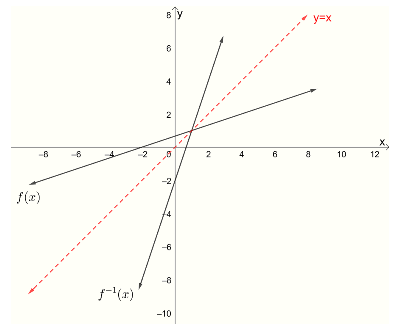
FG3 Inverse Functions



Definition of an Inverse Function

If $f^{-1}(x)$ is the inverse funtion of a one-to-one function $f(x)$ then $f^{-1}(x)$ is the set of all ordered pairs obtained by interchanging the first and second elements in each ordered pair.

So if $(a, b) \in f$ then $(b, a) \in f^{-1}$ and if $f(a) = b$ then $f^{-1}(b) = a$

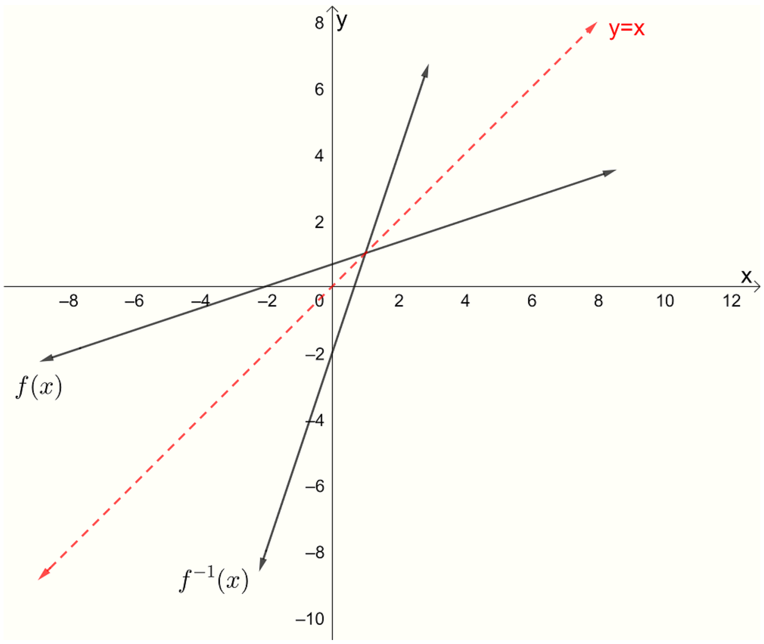
The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}

For example the function $f : R \to R$, defined by $y = f(x) = \frac{x-1}{2}$ has an inverse function with the rule $y = 2x + 1$.

So $(3, 1)$ belongs to f and $(1, 3)$ belongs to f^{-1} and $(-7, -4)$ belongs to f and $(-4, -7)$ belongs to f^{-1} .

Graph of an Inverse Function

The graphs of any one-to-one function, f , and its inverse, f^{-1} , are symmetric about the line $y = x$.



Finding an Inverse Function for $y = f(x)$

To obtain the rule for an inverse function, swap the x and y coordinates in f and rearrange to express y in terms of x .

Example

Find the inverse function of f where $f(x) = 2 - 3x$

$$\begin{aligned} y &= 2 - 3x \\ x &= 2 - 3y \quad \text{(swap the } x \text{ and } y\text{)} \\ x - 2 &= -3y \quad \text{(rearrange to make } y \text{ the subject)} \\ -x + 2 &= 3y \\ \frac{-x+2}{3} &= y \\ f^{-1}(x) &= \frac{-x+2}{3} \end{aligned}$$

Exercise

Find the inverse of each of the following one-to-one functions:

1) $y = x + 5$

2) $y = 4x$

3) $y = \frac{2x+1}{3}$

4) $y = \sqrt{2x-1}$, $x \geq \frac{1}{2}$

Answers

1) $f^{-1}(x) = x - 5$
2) $f^{-1}(x) = \frac{x}{4}$
3) $f^{-1}(x) = \frac{3x-1}{2}$
4) $f^{-1}(x) = \frac{x^2+1}{2}$, $x \geq 0$