

ES3 Quadratic Equations

This tip sheet looks at solutions to quadratic equations using the “null factor law”.

General Form

A *quadratic* equation can be rearranged to the form: $ax^2 + bx + c = 0$ where $a \neq 0$.

Examples

(using $ax^2 + bx + c = 0$ determine a , b and c)

1. $5x^2 - 3x + 9 = 0$ $a = 5$, $b = -3$, $c = 9$

2. $x^2 = 5x - 4 \Rightarrow x^2 - 5x + 4 = 0$ $a = 1$, $b = -5$, $c = 4$

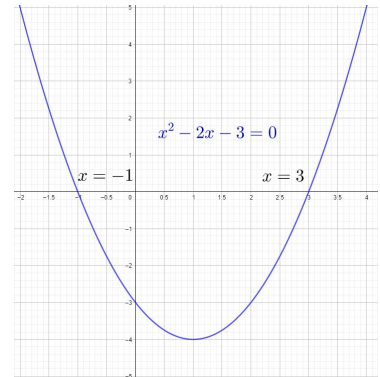
3. $x = \frac{3}{2x} \Rightarrow 2x^2 = 3 \Rightarrow 2x^2 - 3 = 0$ $a = 1$, $b = 0$,
 $c = -3$

Factorisation

If the equation can be factorised then the ‘null factor law’ can be used to find the solutions:

Null factor law uses the simple idea:
If $m \times n = 0$, then $m = 0$ and / or $n = 0$

If the product of two or more factors is zero then any one of the individual factors may be zero and provide a solution for the equation.



The solution to a quadratic equation gives the x - intercepts of its graph.

Example 1 (find all possible values of y)

$$y^2 = 5y$$

$$y^2 - 5y = 0 \text{ (rearrange to form)}$$

$$y(y - 5) = 0 \text{ (rearrange)}$$

for this to be true

$$y = 0 \text{ or } (y - 5) \text{ must equal zero, (null factor law)}$$

$$\therefore \mathbf{y = 0 \text{ or } y = 5}$$

check by substituting back to $y^2 = 5y$

$$\text{If } y = 0, 0^2 = 5 \times 0 \text{ (checked)}$$

$$\text{If } y = 5, 5^2 = 5 \times 5 \text{ (checked)}$$

$$y^2 = 5y$$

Left Hand Side = Right Hand Side

Example 2 (find all possible values of x)

$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0 \text{ (factorise)}$$

$$(x - 4) = 0 \text{ or } (x - 1) = 0 \text{ (null factor law)}$$

$$\therefore \mathbf{x = 4 \text{ or } x = 1}$$

check by substituting back to $x^2 - 5x + 4 = 0$

$$\text{If } x = 4, 4^2 - 5 \times 4 + 4 = 0 \text{ (checked)}$$

$$\text{If } x = 1, 1^2 - 5 \times 1 + 4 = 0 \text{ (checked)}$$

Example 3 (find all possible values of p)

$$p^2 + 10p + 25 = 0$$

$$(p + 5)(p + 5) = 0 \text{ (factorise)}$$

$$(p + 5) = 0 \text{ (null factor law)}$$

$$\therefore \mathbf{p = -5}$$

check by substituting back to $p^2 + 10p + 25 = 0$

$$\text{If } p = -5 \text{ check } (-5)^2 + 10 \times (-5) + 25 = 0$$

Example 4 (find all possible values of m)

$$4m^2 - 49 = 0$$

$$(2m + 7)(2m - 7) = 0 \text{ (factorise by difference of squares)}$$

$$(2m + 7) = 0 \text{ or } (2m - 7) = 0 \text{ (null factor law)}$$

$$\therefore m = \frac{-7}{2} \text{ or } m = \frac{7}{2}$$

check by substituting back to $4m^2 - 49 = 0$

Example 5 (find all possible values of x)

$$x = \frac{-6}{1 - 2x} \text{ provided } x \neq 1/2$$

$$x - 2x^2 = -6 \text{ (multiplying both sides by } 1 - 2x)$$

$$2x^2 - x - 6 = 0 \text{ (rearrange to form } ax^2 + bx + c = 0)$$

$$(2x + 3)(x - 2) = 0 \text{ (factorise)}$$

$$\therefore x = \frac{-3}{2} \text{ or } x = 2 \text{ (solving using null factor law)}$$

Exercises

Solve the following quadratic equations:

1. $x^2 - 6x + 8 = 0$
2. $x^2 + 2x - 3 = 0$
3. $2x^2 - 3x - 2 = 0$
4. $6 - z - z^2 = 0$
5. $2x^2 + 7x = 15$
6. $11p = 3(2p^2 + 1)$

Answers

1. $x = 4, \quad x = 2$
2. $x = -3, \quad x = 1$
3. $x = -1/2, \quad x = 2$
4. $z = -3, \quad z = 2$
5. $x = 3/2, \quad x = -5$
6. $p = 1/3, \quad p = 3/2$