

PME1.1: DIMENSIONS

In the SI system (metres, kilograms, seconds) the unit of speed is the metre per second. This is not the only unit that can be used. Some alternative, equally correct units are mile per hour, kilometre per hour, centimetre per minute, etc. All of these have one thing in common: each is a unit of length per unit of *time*.

*The quantity length per time is called the **dimension** of the property speed.*

When writing the dimensions of a physical quantity we use the symbols [L], [M], [T], and [I] to represent the dimensions of length, mass, time and current respectively. Hence we can write:

$$[\text{speed}] = [\text{length}] [\text{time}]^{-1} = [\text{L}] [\text{T}]^{-1}$$

where the brackets are read as 'dimensions of'.

Example 1

$$[\text{area}] = [\text{L}]^2$$

$$[\text{volume}] = [\text{L}]^3$$

$$[\text{density}] = [\text{M}] [\text{L}]^{-3}$$

The unit of all derived quantities can be found by substituting the fundamental unit for each dimension in the dimension equation. For example, in the SI system the unit of mass is the kilogram (kg) and the unit of length is the metre (m). The unit of density – dimension [M] [L]⁻³ – then becomes kgm⁻³ in the SI system.

Checking Dimensions

The examination of the dimensions of an expression can be used to check the feasibility of given equations. The dimensions of both the left hand side and right hand side of all equations must be dimensionally correct and equal, so that they balance.

Example 2

The equation for the circumference of a circle:

$$C = 2 \pi r$$

Left Hand Side: [circumference] = [L] a length

Right Hand Side: [2 π r] = [radius] **Note:** constant 2 π has no dimension.
 = [L] a length.

Hence the Left Hand side = Right Hand side, therefore the equation is dimensionally correct.

Exercise

Check that the following equations are dimensionally correct:

- (a) $V = \pi r^2 h$ (the volume of a cylinder)
- (b) $A = 4\pi r^2$ (the surface area of a sphere)
- (c) $A = 2\pi r^2 + 2\pi r h$ (the surface area of a cylinder)