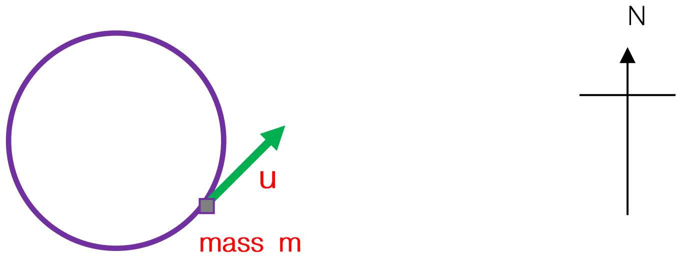
RMIT UNIVERSITY

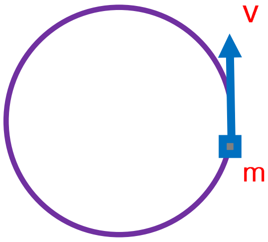
ENST1.3: CIRCULAR MOTION

Centripetal Acceleration

Consider a mass m travelling at a constant speed in a horizontal circle. In the position shown below the instantaneous velocity is shown as a vector u in a direction North East.

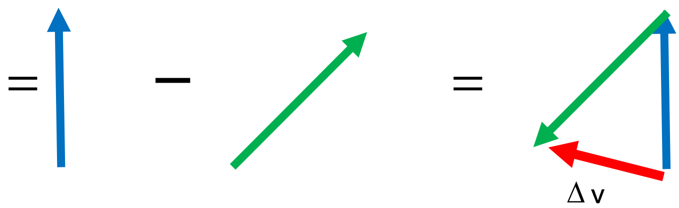


If we look at this mass a short time later it will look like that shown in the diagram below. The instantaneous velocity is now v in a direction North.



Thus, the change in velocity Δv is given by

$\Delta v = v - u$



The change in velocity Δv is shown above. Note that **Δv** must always act radially inwards. Therefore, the mass is accelerating even though the speed is not changing. This acceleration is called centripetal (centre-seeking) acceleration, indicating that the acceleration always acts towards the centre of the circle.

Centripetal acceleration describes the change in velocity with time for an object moving in a circular path at constant speed. The acceleration is given by:

$$\boxed{a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2\, r\, f^2}$$

a = centripetal acceleration (ms^{-2}), v = speed (m/s). Note: $v = (2\times\pi\times r)/T$ or Circumference Travelled ÷Time
r = radius of circle (m), T = period of motion (s)
f = frequency of motion (Hz) Note: $f = 1/T$

Centripetal force

As stated in Newton's 2nd Law of Motion, the net or unbalanced force on an object is given by $\Sigma F = m \times a$, where the net force is always in the same direction as the acceleration. Hence, the net force – as with the centripetal acceleration – acts towards the centre of the circle. This net force is called the centripetal force F_c. Given that $\Sigma F = m \times a$

$$F_C = m \times a = m \times \frac{v^2}{r} = m \times \frac{4\pi^2 r}{T^2} = m \times 4\pi^2\ r\ f^2$$

Using our relationships between linear and angular motion we have:

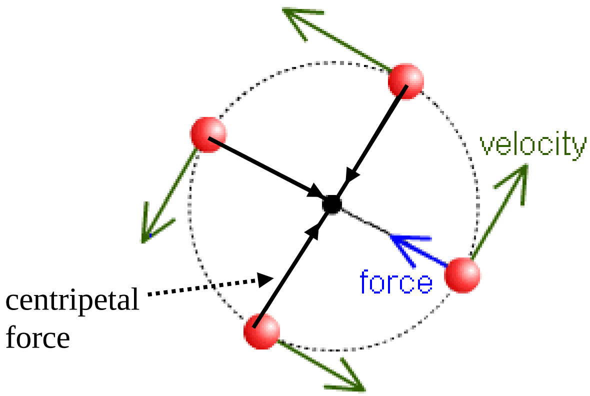
$v = r \times \omega$ Therefore:

$$F_C = m \times \frac{v^2}{r} = m \times \frac{r^2 \omega^2}{r} = m\ r\ \omega^2$$

In angular terms:

$$F_C = m\ r\ \omega^2$$

An object travelling in a circle at constant speed will have a centripetal force that always acts towards the centre (see below), while the velocity is tangential to the circle. Note here that the force is always at right angles to the velocity. In the case of the hammer thrower (see example on next page) the centripetal force is being provided by the thrower at the centre of the circle. This force is a tensional force.



Forces that cause circular motion

In the hammer thrower example below the centripetal force was provided by an inwards tension that the thrower was exerting on the hammer. There are other forces that cause circular motion. Always try to identify the force causing the object to turn in a circle.

For example, a car moving in a circle does so because the centripetal force is due to the friction between the tyres and the road.

The centripetal force causing a satellite (such as the Earth's natural satellite, the moon) to orbit the Earth is caused by the gravitational force of the Earth acting on the satellite.

Example

(Millar, G, et. al, 2000, *Physics 12,* Heinemann)

An athlete swings a 2.5kg hammer in a horizontal circle of radius 0.8m at 2.0 revolutions per second. The wire is horizontal all the time.

(a) What is the period of rotation of the ball?
(b) What is the orbital speed of the ball?
(c) Calculate the acceleration of the ball?
(d) What is the magnitude of the net force acting on the ball?
(e) Name the force that is responsible for the centripetal acceleration of the ball?
(f) Describe the motion of the ball if the wire breaks?

Solution

(a) $T = 1/f$ where f is the frequency of the hammer (2 rev s^{-1})
$= 1/2.0\ s^{-1} = 0.50\ s$

(b) $v = (2 \times \pi \times r)/T = 2\pi fr$ Recall: $f = 1/T$
$= 2\pi(2.0\ Hz)(0.80\ m) = 10.1\ m\ s^{-1}$

(c) $a = v^2/r$
$= (10.053\ m\ s^{-1})^2/0.80\ m = 126\ m\ s^{-2}$ towards centre of circle

(d) $F = ma$
$= (2.5\ kg)(12.6\ m\ s^{-2}) = 316\ N$

(e) The force that is responsible for the centripetal acceleration of the ball is the tension in the wire, which is directed radially inwards at all times.

(f) If the wire breaks, the ball will move off at a tangent to the circle with a speed of 10.1 m s^{-1}.

Exercise

An ice skater of mass 50 kg is skating in a horizontal circle of readius 1.5m at a constant speed of 2.0 m/s.

(a) What is the acceleration of the skater?
(b) What is the horizontal component of the centripetal force acting on the skater?
(c) Name the force that is providing the horizontal component of the net force enabling the skater to move in a circular path?

Answers

(a) $a = v^2/r$
$= (2.0\ m\ s^{-1})^2/1.5\ m$
$= 2.7\ m\ s^{-2}$ towards centre of circle

(b) $F = ma$
$= (50\ kg)(2.67\ m\ s^{-2})$
$= 133\ N$ towards centre of circle

(c) The horizontal force exerted on the blades by the ice; i.e. friction