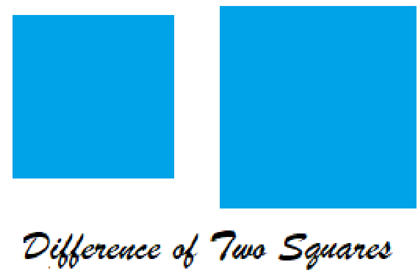
RMIT UNIVERSITY

A3.3 Factorisation: Difference of Two S



The difference of two squares formula is commonly used in mathematics. It allows us to factorise terms such as

$$x^2 - 36,$$
$$5x^2 - 20y^2,$$
$$a^2 - b^2.$$

This module explains the formula and shows how it may be used to factorise algebraic expressions.

It is an important rule that you should commit to memory.

Factorisation: Difference of Two Squares (DOTS).

Consider the following expansion:

$$\begin{aligned}(x+5)(x-5) &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25 \\ &= x^2 - 5^2\end{aligned}$$

In general,

$$\begin{aligned}(x+a)(x-a) &= x^2 - ax + ax - a^2 \\ &= x^2 - a^2.\end{aligned}$$

Note that:

- The terms in the brackets differ only in the sign of the second term
- The expansion is the difference of two terms, both of which are perfect squares

More generally:

$$(a+b)(a-b) = a^2 - b^2$$

so

$$a^2 - b^2 = (a+b)(a-b),$$

and is known as the DOTS (Difference Of Two Squares) rule.

This can be used to factorise expressions of the form $a^2 - b^2$.

Watch a short video on factorising using " Difference of two squares"

Download a transcript of the video on " Difference of two squares"

Examples:

1. Factorise $a^2 - 36$.
Solution:

$$\begin{aligned} a^2 - 36 &= a^2 - (6)^2 \text{ expression is the difference of two squares} \\ &= (a+6)(a-6) \text{ using the DOTS rule} \\ &= (a-6)(a+6) \text{ order doesn't matter, either is correct.} \end{aligned}$$

2. Factorise $4^2 - y^2$.
Solution:

$$\begin{aligned} 4^2 - y^2 &= (2)^2 - y^2 \text{ expression is the difference of two squares} \\ &= (2+y)(2-y) \text{ using the DOTS rule.} \end{aligned}$$

3. Factorise $3x^2 - 48$.
Solution:
At first sight, we cannot use DOTS. But taking out a common factor of 3 we have

$$\begin{aligned} 3x^2 - 48 &= 3\left(x^2 - 16\right) \\ &= 3\left(x^2 - 4^2\right) \\ &= 3(x+4)(x-4) \text{ using the DOTS rule.} \end{aligned}$$

4. Factorise $(x+2)^2 - 9$.
Solution:

$$(x+2)^2 - 9 = (x+2)^2 - 3^2 \text{ expression is the difference of two squares}$$
$$= (x+2+3)(x+2-3) \text{ using the DOTS rule}$$
$$= (x+5)(x-1).$$

5. Factorise $y^2 + 36$.
Solution:
This expression is the sum of two squares, not the difference hence the DOTS rule cannot be applied. There are no real factors for this expression. [1]

[1] There are complex factors but we do not consider them in this module.

Exercise

Factorise the following expressions using the DOTS rule (if possible):
a. $x^2 - 4$ b. $a^2 - 100$ c. $49 - x^2$ d. $64x^2 - 1$
e. $121x^2 - 49y^2$ f. $a^2b^2 - 25$ g. $5x^2 - 20$ h. $a^2 + 100$
i. $x^2y^3 - 36y$ j. $(x+2)^2 - y^2$ k. $(x-5)^2 - 36$ l. $(a+1)^2 - (b-2)^2$

Answers

a. $(x+2)(x-2)$ b. $(a+10)(a-10)$ c. $(7+x)(7-x)$
d. $(8x+1)(8x-1)$ e. $(11x+7y)(11x-7y)$ f. $(ab+5)(ab-5)$
g. $5(x+2)(x-2)$ h. Does not factorise. i. $y(xy+6)(xy-6)$
j. $(x+2+y)(x+2-y)$ k. $(x-11)(x+1)$ l. $(a+b-1)(a-b+3)$.