A2.1 Rearranging Formulae

Rearranging formulas (also called transposing of formulas) is a necessary skill for many courses. This module looks at some essential skills required before you move on to more complicated examples.

Play a short video.

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Introduction

Some of the most important equations that we might be required to transpose occur frequently in science, engineering and economics. They are called formulae and give a general rule describing the relationship between variable quantities

Here are some examples:

$$A = \pi r^2$$

$$s = ut + \frac{1}{2}at^2$$

$$S = P(1+i)^n$$

In these examples A, s and S are, respectively, the subjects[1] of the formulae. Sometimes a formula is given in a particular form and it is necessary to rearrange the formula to make a different variable the subject:

We know the area of a circle $A = \pi r^2$ where r is the radius and so we can calculate A for any value of r. What if we know the area A and have to calculate the radius r?[2]

We know $s = ut + \frac{1}{2}at^2$ but what if we know s and t and want to calculate a?[3]

[1] A, s and S are called subjects because they are on the left hand side of the formula and followed by an equal sign.

[2] We need r to be the subject rather than A.

[3] We need a to be the subject rather than s.

Basic Rule

> When rearranging equations and formulas, whatever you do on one side of the equal sign, you must do on the other.

Examples

1. Make A the subject in the formula $A + B = C$.
   Solution:[4]

$$\begin{aligned} A + B &= C \\ A + B - B &= C - B \quad \text{subtracting } B \text{ from both sides} \\ A &= C - B. \end{aligned}$$

[4] We have to get rid of the B term on the left hand side to get A on its own. The B is being added to A on the left side so we subtract B from the left side to get rid of the B terms on the left. Our rule says we must subtract the B from the right side as well.

2. Make A the subject in the formula $A - B = C$.
   Solution:[5]

$$\begin{aligned} A - B &= C \\ A - B + B &= C + B \quad \text{adding } B \text{ to both sides} \\ A &= C + B. \end{aligned}$$

[5] We have to get rid of the B term on the left hand side to get A on its own. The B is being subtracted from A on the left side so we add B to the left side to get rid of it. Our rule says we must add B to the right side as well.

3. Make A the subject in the formula $AB = C$.
   Solution:[6]

$$\begin{aligned} AB &= C \\ \frac{AB}{B} &= \frac{C}{B} \quad \text{dividing both sides by } B \\ A &= \frac{C}{B} \quad \text{cancelling the } B\text{'s on the left side.} \end{aligned}$$

[6] We have to get rid of the B term on the left hand side to get A on its own. The B is multiplying A on the left side so we divide by B on the left side to get rid of it. Our rule says we must divide by B on the right side as well.

4. Make A the subject in the formula $\frac{A}{B} = C$.
   Solution:[7]

$$\begin{aligned} \frac{A}{B} &= C \\ \frac{AB}{B} &= BC \quad \text{multiplying both sides by } B \\ A &= BC \quad \text{cancelling the } B\text{'s on the left side.} \end{aligned}$$

[7] We have to get rid of the B term on the left hand side to get A on its own. The B is dividing A on the left side so we multiply by B on the left side to get rid of it. Our rule says we must multiply by B on the right side as well.

5. Make A the subject of the formula $A^2 = B$.
   Solution:[8]

$$\begin{aligned} A^2 &= B \\ \sqrt{A^2} &= \sqrt{B} \quad \text{taking the square root of both sides} \\ A &= \sqrt{B}. \end{aligned}$$

[8] This case involves A^2. The inverse (opposite) operation to squaring is the square root. So to get A on its own we need to take the square root of both sides.

6. Make A the subject of the formula $\sqrt{A} = B$.
   Solution:[9]

$$\sqrt{A} = B$$
$$\left(\sqrt{A}\right)^2 = B^2$$
$$A = B^2.$$

[9] This case involves $\sqrt{A}$. The inverse (opposite) operation to the square root is squaring. So to get A on its own we need to square both sides.

Inverse operations

In the above examples we used *inverse* operations to " undo" operations. Remember:

subtraction undoes addition	conversely	addition undoes subtraction
division undoes multiplication	conversely	multiplication undoes division
square root undoes square	conversely	square undoes square root
$\sqrt[n]{x}$ undoes x^n	conversely	x^n undoes $\sqrt[n]{x}$

Also remember:

$B + C = A$ is the same as $A = B + C$ and
$\sqrt{A^2} = A$ and $(\sqrt{B})^2$= B. For example: $\sqrt{3^2} = 3$ and $\left(\sqrt{25}\right)^2$= 25.

Examples:

1. Transform $V = A - K$ to make A the subject
   Solution:

$$\begin{aligned} V &= A - K \quad \text{(we want } A \text{ to be the subject)} \\ V + K &= A - K + K \quad \text{(add } K \text{ to both sides)} \\ V + K &= A \\ A &= V + K \quad \text{(making } A \text{ the subject)} \end{aligned}$$

2. Make d the subject of $C = \pi d$

$$\begin{aligned} C &= \pi d \quad \text{(we want } d \text{ to be the subject)} \\ \frac{C}{\pi} &= \frac{\pi d}{\pi} \quad \text{(divide both sides by } \pi \text{ then cancelling)} \\ \frac{C}{\pi} &= d \\ d &= \frac{C}{\pi} \quad \text{(making } d \text{ the subject)} \end{aligned}$$

3. Rearrange $j = 3w - 5$ in terms of w .

$$j = 3w–5 \quad \text{(we want } w \text{ to be the subject)}$$
$$j + 5 = 3w–5 + 5 \quad \text{(add 5 to both sides)}$$
$$j + 5 = 3w \quad \text{(giving } 3w \text{ as the subject)}$$
$$\frac{j+5}{3} = \frac{3w}{3} \quad \text{(divide both sides by 3 then cancelling)}$$
$$\frac{j+5}{3} = w$$
$$w = \frac{j+5}{3} \quad \text{(making } w \text{ the subject)}$$

4. Make c the subject of $E = mc^2$.

$$E = mc^2 \quad \text{(we want } c \text{ to be the subject)}$$
$$\frac{E}{m} = \frac{mc^2}{m} \quad \text{(divide both sides by } m\text{)}$$
$$\frac{E}{m} = c^2 \quad \text{(cancelling)}$$
$$\sqrt{\frac{E}{m}} = \sqrt{c^2} \quad \text{(square root both sides)}$$
$$\sqrt{\frac{E}{m}} = c \quad \text{(remember } \sqrt{3^2} = 3\text{)}$$
$$c = \sqrt{\frac{E}{m}} \quad \text{(rearanging making } c \text{ the subject)}$$

Exercises:

1. $m = n - 2$ Find n
2. $A = 2B + C$ Find C
3. $A = 2B + C$ Find B
4. $P = \frac{k}{v}$ Find K
5. $PV = k$ Find V
6. $v = u + at$ Find a
7. $v = u + at$ Find t
8. $r = \sqrt{\frac{A}{\pi}}$ Find A
9. $A = x^2$ Find x
10. $A = \pi r^2$ Find r

Answers:

1. $n = m + 2$
2. $C = A - 2B$
3. $B = \frac{A-C}{2}$
4. $k = PV$
5. $V = \frac{K}{P}$
6. $a = \frac{V-U}{t}$
7. $t = \frac{V-U}{a}$
8. $A = \pi r^2$
9. $x = \pm\sqrt{A}$
10. $r = \pm\sqrt{\frac{A}{\pi}}$