A1.5 Algebraic Fractions: Multiplication and Division

$(x^2 - y^2)/(x+y) =$
$2x+4 = 2(x+$
$a^2-b^2 = (a-b)$

This module is about multiplication and division of fractions containing algebraic terms. Multiplication and division of numerical fractions is revised very briefly.

Simplifying Fractions

You have probably seen numerical fractions like $\frac{18}{24}$ before. Generally we like to reduce fractions to their simplest form. This is done by dividing the numerator and denominator[1] by the same number. So

$$\frac{18}{24} = \frac{9}{12} \quad \text{dividing top and bottom numbers by two}$$
$$= \frac{3}{4} \quad \text{dividing top and bottom numbers by three}$$

this is the simplest form as there is no number that divides into 3 and 4. Note that this can be done in one step if you realise that 6 divides into 18 and 24. Then you have

$$\frac{18}{24} = \frac{3}{4} \quad \text{dividing top and bottom numbers by six.}$$
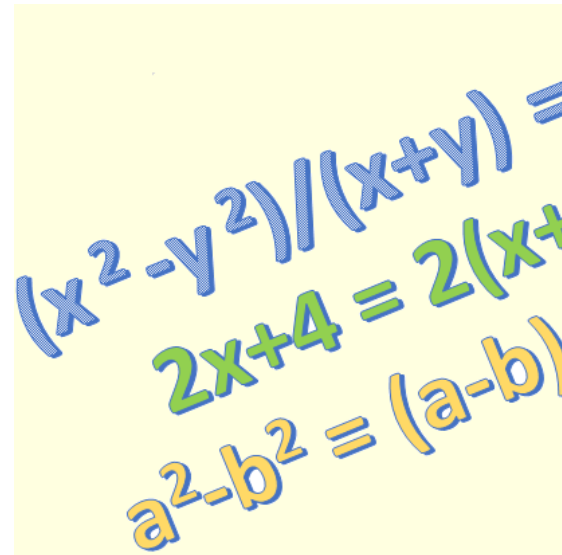
Simplifying fractions by dividing the same number into both the numerator and denominator is called cancelling.

[1] The numerator is the number on top and the denominator is the number at the bottom of the fraction.

Simplifying Algebraic Fractions

We can use the same technique to simplify algebraic fractions as we did in simplifying numerical fractions. For example:

1. Simplify $\frac{4x^2y}{6y^2}$.

$$\frac{4x^2y}{6y^2} = \frac{2x^2y}{3y^2} \quad \text{dividing top and bottom by two}$$
$$= \frac{2x^2}{3y} \quad \text{dividing top and bottom by } y.$$

2. Simplify $\frac{a(b+2c)}{2ab}$.

This example shows it is important to divide all terms on the top by the same number.[2]

$$\frac{a(b+2c)}{2ab} = \frac{(b+2c)}{2b} \quad \text{dividing top and bottom by } a$$
$$= \frac{b+2c}{2b}.$$

3. Simplify $\frac{m-n}{(m-n)^2}$. [3]

$$\frac{m-n}{(m-n)^2} = \frac{m-n}{(m-n)(m-n)} \quad (m \neq n)$$
$$= \frac{1}{m-n} \quad \text{dividing top and bottom by } m-n.$$

4. Simplify $\frac{3x^2y}{6x+9y}$. [4]

$$\frac{3x^2y}{6x+9y} = \frac{3x^2y}{3(2x+3y)} \quad \text{taking out the factor of 3 in the denominator}$$
$$= \frac{x^2y}{2x+3y} \quad \text{dividing top and bottom by 3.}$$

5. Simplify $\frac{p-2}{(6p-3p^2)}$. [5]

$$\frac{p-2}{(6p-3p^2)} = \frac{p-2}{3(2p-p^2)} \quad \text{taking out the factor of 3 in the denominator}$$
$$= \frac{p-2}{3p(2-p)} \quad \text{taking out the factor of } p \text{ in the denominator}$$
$$= -\frac{p-2}{3p(p-2)} \quad \text{using } p-2 = -(2-p)$$
$$= -\frac{1}{3p} \quad \text{dividing top and bottom by } p-2.$$

Note that you can only divide the denominator and numerator if ALL the terms have a common factor. For example, it would be WRONG to write

$$\frac{x+2}{2y} = \frac{x+1}{y}$$

because the x in the top line has not been divided by 2.

To practice these techniques, please look at Exercise 1.

[2] Note that you cannot divide the $2c$ on top and the $2b$ on the bottom by 2. This is because all the terms on the top and bottom have to be divided by 2. So it would be wrong to write

$$\frac{b+2c}{2b} = \frac{b+c}{b}$$

because the b on the top line was not divided by 2.

[3] This example uses the fact that $(m-n)^2 = (m-n)(m-n)$. Since m and n are just numbers, $m-n$ is also a number and so we can divide by $m-n$ provided that $m \neq n$. If $m = n$, $m-n = 0$ and we mustn't divide anything by zero.

[4] This example factorizes the denominator first. We then divide the top and bottom of the fraction by the common factor 3.

[5] This example uses the fact that $p-2 = -(2-p)$.

Multiplying Fractions

When we multiply two fractions, we multiply the numerators and the denominators of each fraction. For example,

$$\frac{3}{4} \times \frac{7}{5} = \frac{21}{20}.$$

It is best to simplify each fraction, if possible, before you do the multiplication. For example,

$$
\begin{aligned}
\frac{15}{8} \times \frac{24}{35} &= \frac{15}{1} \times \frac{3}{35} && \text{dividing the 24 in the top line and the 8 in the bottom line by 8} \\
&= \frac{3}{1} \times \frac{3}{7} && \text{dividing the 15 in the top line and the 35 in the bottom line by 5} \\
&= \frac{9}{7}.
\end{aligned}
$$

Multiplying Algebraic Fractions

You can also use these ideas with algebraic fractions. For example,

$$
\begin{aligned}
\frac{5a}{7} \times \frac{14}{a} &= \frac{5}{7} \times \frac{14}{1} && \text{dividing the } 5a \text{ in the top line and the } a \text{ in the bottom line by } a \\
&= \frac{5}{1} \times \frac{2}{1} && \text{dividing the 14 in the top line and the 7 in the bottom line by 7} \\
&= \frac{10}{1} \\
&= 10.
\end{aligned}
$$

Here are some more examples:[6]

1. Simplify $\frac{x}{6(x-2)} \times \frac{3(x-2)}{x^2}$.

$$
\begin{aligned}
\frac{x}{6(x-2)} \times \frac{3(x-2)}{x^2} &= \frac{x}{6} \times \frac{3}{x^2} && \text{dividing the top and bottom lines by } x-2 \\
&= \frac{x}{2} \times \frac{1}{x^2} && \text{dividing the top and bottom lines by } 3 \\
&= \frac{1}{2} \times \frac{1}{x} && \text{dividing the top and bottom lines by } x \\
&= \frac{1}{2x}.
\end{aligned}
$$

[6] Note that we are showing every step in the examples below and so the solutions may appear long and complicated. You don't have to do this. You can take as many steps as you like. As you get more familiar with algebra you will naturally use fewer steps to get a result.

2. Simplify $\frac{3m+12}{10} \times \frac{5}{m^2+4m}$.

$$
\begin{aligned}
\frac{3m+12}{10} \times \frac{5}{m^2+4m} &= \frac{3(m+4)}{10} \times \frac{5}{m^2+4m} \quad \text{taking out a factor of } 3 \\
&= \frac{3(m+4)}{10} \times \frac{5}{m(m+4)} \quad \text{taking out a factor of } m \\
&= \frac{3}{10} \times \frac{5}{m} \quad \text{dividing the top and bottom by } m+4 \\
&= \frac{3}{2} \times \frac{1}{m} \quad \text{dividing the top and bottom lines by } 5 \\
&= \frac{3}{2m}.
\end{aligned}
$$

Dividing Fractions

The reciprocal of a fraction is just the fraction turned upside down. So the reciprocal of $3/4$ is $4/3$ and the reciprocal of $2/7$ is $7/2$. Dividing by a fraction is the same as multiplying by the reciprocal[7]. For example:

[7] When dividing fractions we change the divide sign to times and turn the last fraction upside down.

$$
\begin{aligned}
\frac{5}{4} \div \frac{19}{8} &= \frac{5}{4} \times \frac{8}{19} \\
&= \frac{5}{1} \times \frac{2}{19} \\
&= \frac{10}{19}.
\end{aligned}
$$

Dividing Algebraic Fractions

We use the same technique for dividing algebraic fractions as we use for dividing numerical fractions.

Examples

1. Simplify $\frac{7p}{12} \div \frac{3}{8}$.

$$
\begin{aligned}
\frac{7p}{12} \div \frac{3}{8} &= \frac{7p}{12} \times \frac{8}{3} \quad \text{changing sign and inverting the last fraction} \\
&= \frac{7p}{3} \times \frac{2}{3} \quad \text{dividing top and bottom by } 4 \\
&= \frac{14p}{9}.
\end{aligned}
$$

2. Simplify $\frac{m^2}{n} \div 6m$. [8]

[8] Don't forget that $6m = 6m/1$

$$\frac{m^2}{n} \div 6m = \frac{m^2}{n} \div \frac{6m}{1}$$

$$= \frac{m^2}{n} \times \frac{1}{6m} \quad \text{changing sign and inverting the last fraction}$$

$$= \frac{m}{n} \times \frac{1}{6} \quad \text{dividing top and bottom by } m$$

$$= \frac{m}{6n}.$$

3. Simplify $\frac{4(x+3)}{9} \div \frac{24}{5x}$.

$$\frac{4(x+3)}{9} \div \frac{24}{5x} = \frac{4(x+3)}{9} \times \frac{5x}{24} \quad \text{changing sign and inverting the last fraction}$$

$$= \frac{(x+3)}{9} \times \frac{5x}{6} \quad \text{dividing top and bottom by } 4$$

$$= \frac{5x(x+3)}{54}.$$

4. Simplify $\frac{2a+4}{15} \div \frac{a+2}{6}$.

$$\frac{2a+4}{15} \div \frac{a+2}{6} = \frac{2a+4}{15} \times \frac{6}{a+2}$$

$$= \frac{2(a+2)}{15} \times \frac{6}{a+2}$$

$$= \frac{2}{15} \times \frac{6}{1}$$

$$= \frac{2}{5} \times \frac{2}{1}$$

$$= \frac{4}{5}.$$

Exercise 1

Simplify the following fractions:

1. $\frac{12ab^2}{8bc}$

2. $\frac{5x-20}{5}$

3. $\frac{9u-18}{2u-4}$

4. $\frac{6t-9}{12-8t}$

5. $\frac{b}{b^2+7b}$

6. $\frac{(j+4)(j-4)}{3j+12}$

7. $\frac{2(5-v)}{3v-15}$

8. $\frac{9r^2-3r}{16r-48r^2}$

Exercise 2

A. Simplify

1. $\frac{4}{5} \times \frac{15}{16}$

2. $\frac{4a}{3} \times \frac{9}{a}$

3. $\frac{32h^2}{9j} \times \frac{27j}{48h}$

4. $\frac{3d-2}{3} \times \frac{4}{3d-2}$

5. $\frac{2r+4}{3r-9} \times \frac{5r-15}{7r+14}$

6. $\frac{10p-5}{3} \times \frac{3q+3}{2p-1}$

7. $\frac{4g^2-6g}{8} \times \frac{3}{6g-9}$

8. $\frac{3-2y}{33y-11} \times \frac{18y^2-6y}{7-2y}$

B. Simplify

1. $\frac{4m-16}{m} \div \frac{8m-32}{8m}$

2. $\frac{6xy-5y^2}{4x+10y} \div \frac{12x^2-10xy}{12x+30y}$

Answers

Exercise 1.

1. $3ab/2c$	2. $x-4$	3. $9/2$	4. $-3/4$	
5. $\frac{1}{b+7}$	6. $\frac{j-4}{3}$	7. $-2/3$	8. $-3/16$	

Exercise 2

A1. $3/4$	A2. 12	A3. $2h$	A4. $4/3$
A5. $10/21$	A6. $5(q+1)$	A7. $g/4$	A8. $\frac{6y(3-2y)}{11(7-2y)}$
B1. 4	B2. $3y/2x$		