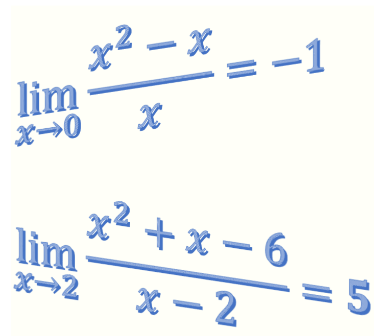
RMIT UNIVERSITY



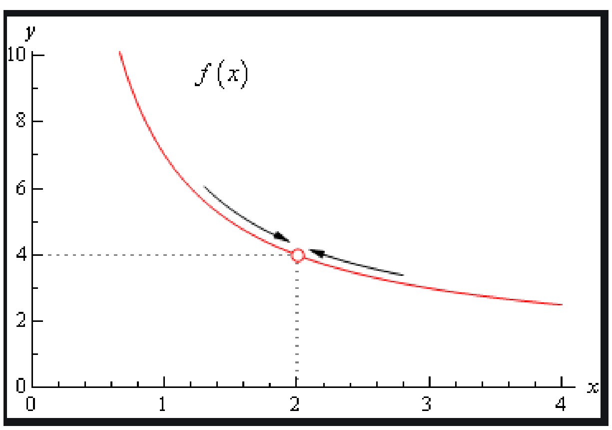
D1 Limits

The limit of a function describes the behaviour of the function as the variable approaches a particular value. We are not concerned with the value of the function at the point of interest, but only what value is being approached.

View an introductory video on limits.

Definition

If the value of the function as we approach a particular value $x = a$ is the same whether we are approaching from the left or the right then the limit exists. The function need not be continuous or even defined at the point of interest. For the function below, the limit of $f(x)$ as x approaches 2 is 4.



We say,

$$\lim_{x \to 2} f(x) = 4.$$

Examples

1. Find the limit of the function $f(x) = x + 2$ as x approaches 2.
   Solution:
   The behaviour of $f(x)$ as $x \to 2$ is shown in the table:

x	1.95	1.995	$\dashrightarrow 2 \dashleftarrow$	2.005	2.05
$f(x)$	3.95	3.995	$\dashrightarrow 4 \dashleftarrow$	4.005	4.05

The table shows that as x approaches 2, $f(x)$ approaches 4

Alternatively, because this function is defined at $x = 2$, we could substitute $x = 2$ into the function:

$$f(2) = 2 + 2 = 4$$

Therefore

$$\lim_{x \to 2} (x + 2) = 4.$$

2. Find $\lim_{h \to 0}(5x^2 + 2xh + h)$.

Solution:

The $2xh$ and h terms will approach zero as h approaches zero. The limit can be found by substituting zero for h:

$$\begin{aligned} \lim_{h \to 0}(5x^2 + 2xh + h) &= 5x^2 + 2 \times 0 + 0 \\ &= 5x^2 \end{aligned}$$

3. Find $\lim_{x \to 1} \frac{x^2-1}{x-1}$.

Solution: It is not possible to find the limit by substituting $x = 1$ as we would then have 0 in the denominator. We can find this limit, if we simplify the expression first[1] and then substitute.

In this case

$$\begin{aligned} \lim_{x \to 1} \frac{x^2-1}{x-1} &= \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} \\ &= \lim_{x \to 1} x + 1 \\ &= 2. \end{aligned}$$

[1] You need to know the difference of two squares,

$$a^2 - b^2 = (a - b)(a + b).$$

Applying this gives:

$$x^2 - 1 = (x - 1)(x + 1).$$

4. Find $\lim_{h \to 0} \sin(h) / h$.

Solution:

It is not possible to find the limit by substituting $h = 0$. But consider the behaviour of $f(h) = \frac{\sin(h)}{h}$ as $h \to 0$:

h	-0.5	-0.3	-0.2	-0.1	0	0.01	0.1	0.2	0.3	0.5
$f(h)$	0.959	0.985	0.993	0.998	?	0.99998	0.998	0.993	0.985	0.959

It appears that

$$\lim_{h \to 0} \frac{\sin(h)}{h} = 1.$$

Exercises

Determine the following limits:

1) $\lim_{x \to 0} (4xh + 2)$
2) $\lim_{x \to 0} \frac{9-x^2}{4}$
3) $\lim_{h \to 0} \frac{xh-2h}{h}$
4) $\lim_{x \to 0} \frac{\tan(x)}{x}$

Answers

1) 2
2) 9/4
3) $x - 2$
4) 1