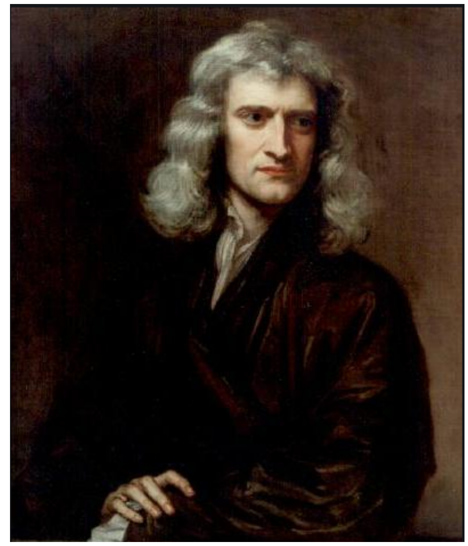
Antidifferentiation

Antidifferentiation (also called integration) is the opposite operation to differentiation. Given a derivative $f'(x)$ of a function we want to find the original function $f(x)$. The original function is called an antiderivative.

Play a short video on Antidifferentiation.



Isaac Newton co-invented Calculus which comprises differentiation and antidifferentiation (integration). This portrait of Newton at age 46 was done by Godfrey Kneller in 1689. (https://en.wikipedia.org/wiki/Isaac_Newton)

The Basic Concept

Think about the following examples:

1. If $f(x) = \frac{x^3}{3}$, then $f'(x) = x^2$ so $\frac{x^3}{3}$ is an antiderivative of x^2.

2. If $f(x) = \frac{x^3}{3} + 1$, then $f'(x) = x^2$ so $\frac{x^3}{3} + 1$ is an antiderivative of x^2.

3. If $f(x) = \frac{x^3}{3} + 2$, then $f'(x) = x^2$ so $\frac{x^3}{3} + 2$ is an antiderivative of x^2.

Notice that adding a constant to $\frac{x^3}{3}$ does not change the fact it is an antiderivative of x^2. This is because the derivative of a constant is zero.

In general an antiderivative of $f'(x)$ is given by $f(x) + c$ where c is a constant[1].

[1] We often write $c \in \mathbb{R}$ which means that c is a real number.

Finding Antiderivatives

Antidifferentiation is more complicated than differentiation. However there are some rules to help us. One of the most important is the power rule which says[2]:

> If $f'(x) = x^n, n \neq -1$ the antiderivative $f(x) = \frac{1}{n+1}x^{n+1} + c$, where c is a constant.

[2] Note that if $n = -1$, $1/n+1$ would be $1/0$ which has no meaning. Apart from this restriction, n can be any number.

Alternate Notation

If $y = f(x)$ then $dy/dx = f'(x)$. If $dy/dx = x^n$, the antiderivative $y = \frac{1}{n+1}x^{n+1} + c$, where c is a constant.

Examples

1. Given $dy/dx = x$,find the antiderivative. [3]

$$y = \frac{x^{1+1}}{1+1} + c,\ c \in \mathbb{R} \quad \text{(add one to the power of } x\text{, divide by the new power and add a constant)}$$
$$= \frac{x^2}{2} + c.$$

[3] In this case $n = 1$ because $x = x^1$.

2. Given $dy/dx = 1$,find the antiderivative. [4]

$$y = \frac{x^{0+1}}{0+1} + c,\ c \in \mathbb{R} \quad \text{(add one to the power of } x\text{, divide by the new power and add a constant)}$$
$$= x + c.$$

[4] In this case $n = 0$ because $x^0 = 1$.

3. Given $dy/dx = x^{-3}$,find the antiderivative. [5]

$$y = \frac{x^{-3+1}}{-3+1} + c,\ c \in \mathbb{R}$$
$$= \frac{x^{-2}}{-2} + c$$
$$= -\frac{1}{2x^2} + c.$$

[5] In this case $n = -3$. The new power will be -2.

4. Given $dy/dx = \sqrt{x}$,find the antiderivative.[6]

$$y = \frac{x^{1/2+1}}{1/2+1} + c,\ c \in \mathbb{R}$$
$$= \frac{x^{3/2}}{3/2} + c$$
$$= \frac{2}{3}x^{3/2} + c.$$

[6] In this case, $n = 1/2$ because $\sqrt{x} = x^{1/2}$. The new power will be $1/2 + 1 = 3/2$.

Exercises

Find antiderivatives for the following:
1. x^3 2. s^8 3. $\sqrt[3]{x}$ 4. x^{-5} 5. 6 6. m^{-2} 7. $p^{-1/2}$

Answers

In all cases, c is a constant.
1. $\frac{1}{4}x^4 + c$ 2. $\frac{1}{9}s^9 + c$ 3. $\frac{3}{4}x^{4/3} + c$ 4. $-\frac{1}{4}x^{-4} + c$ 5. $6x + c$ 6. $-m^{-1} + c = -\frac{1}{m} + c$ 7. $2p^{1/2} + c$